MHD

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Why Plasma Physics and MHD?

Most of ordinary matter in the Universe is in gaseous or liquid form that is either ionized or in a state that conducts electricity

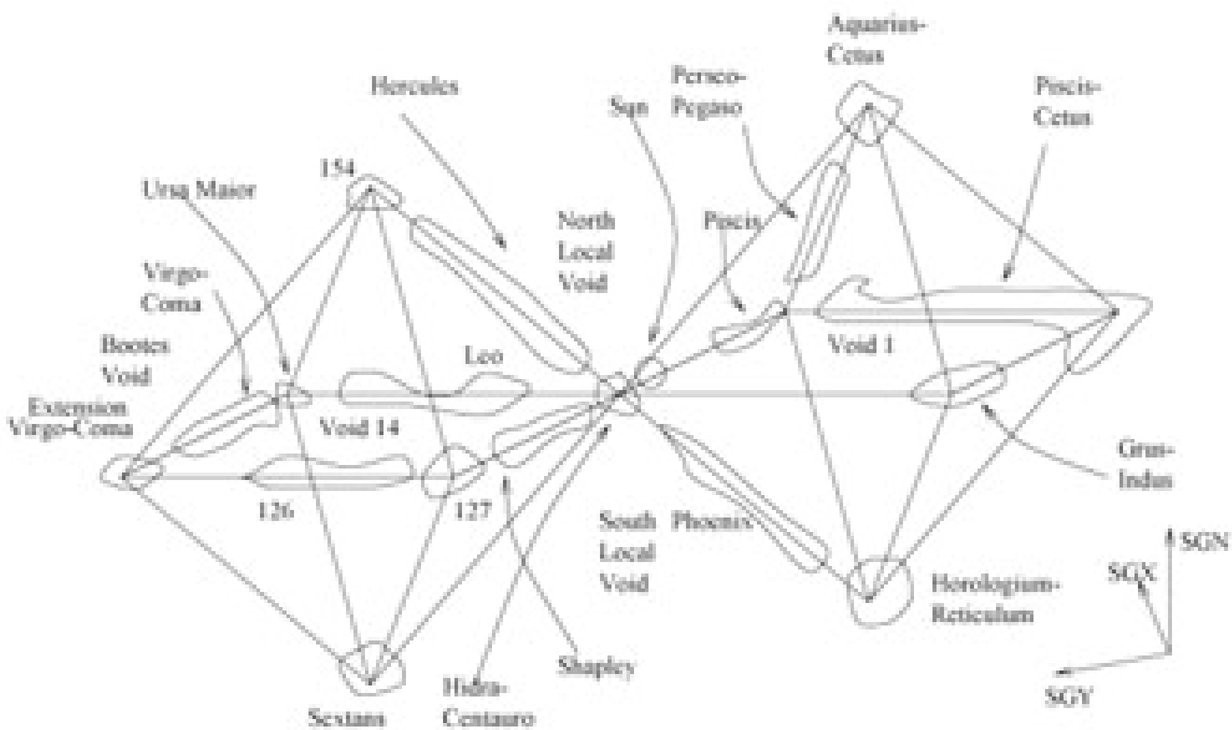
Most of this matter is magnetized

What are the origins and consequences of this?

Outline

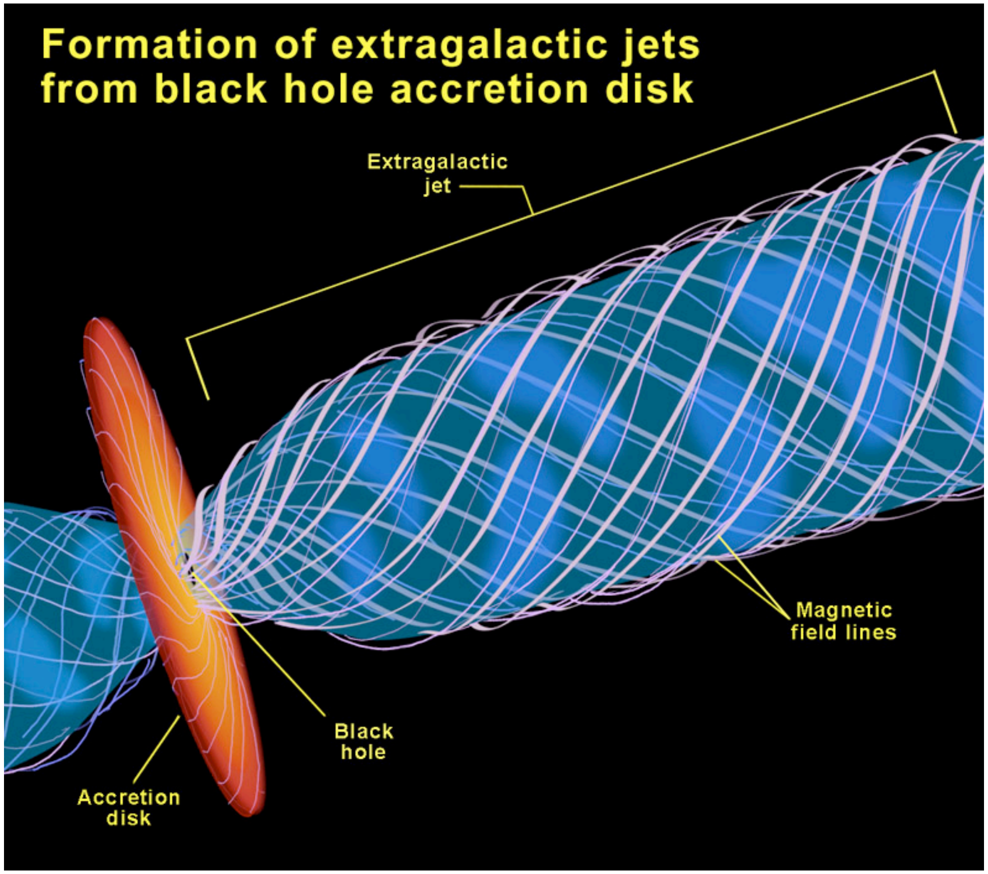
- Examples of physical systems
- Governing equations
- Basic properties of these equations
- Numerical methods to solve them
- Examples of applications
- References and resources

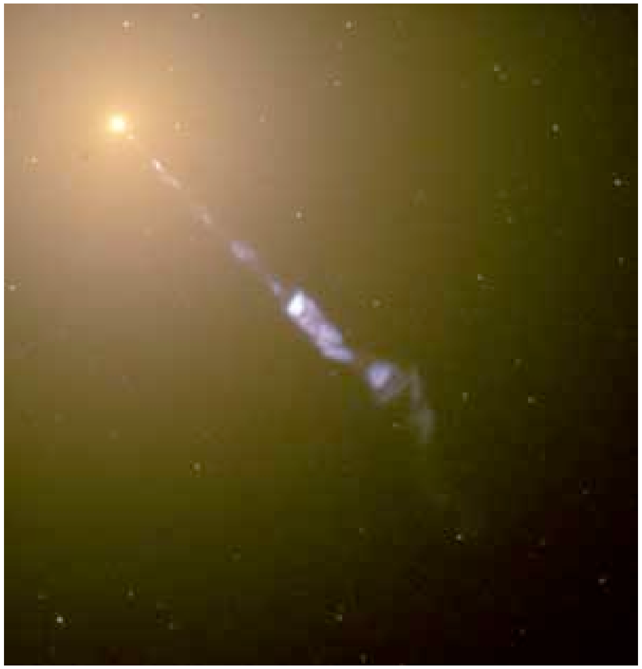
Large-Scale Structure in the Universe



Credit: Battaner

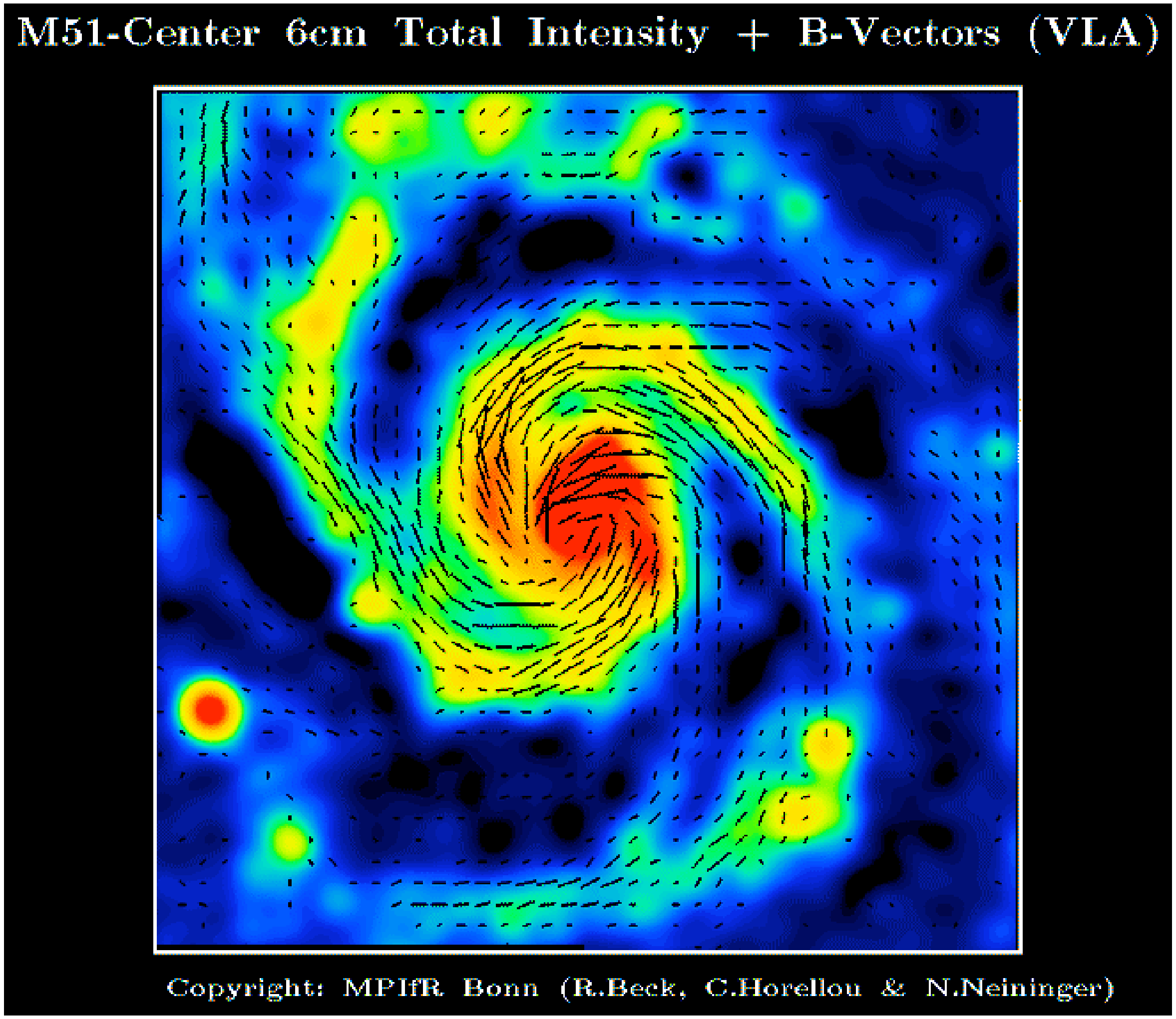
Extragalactic Jets



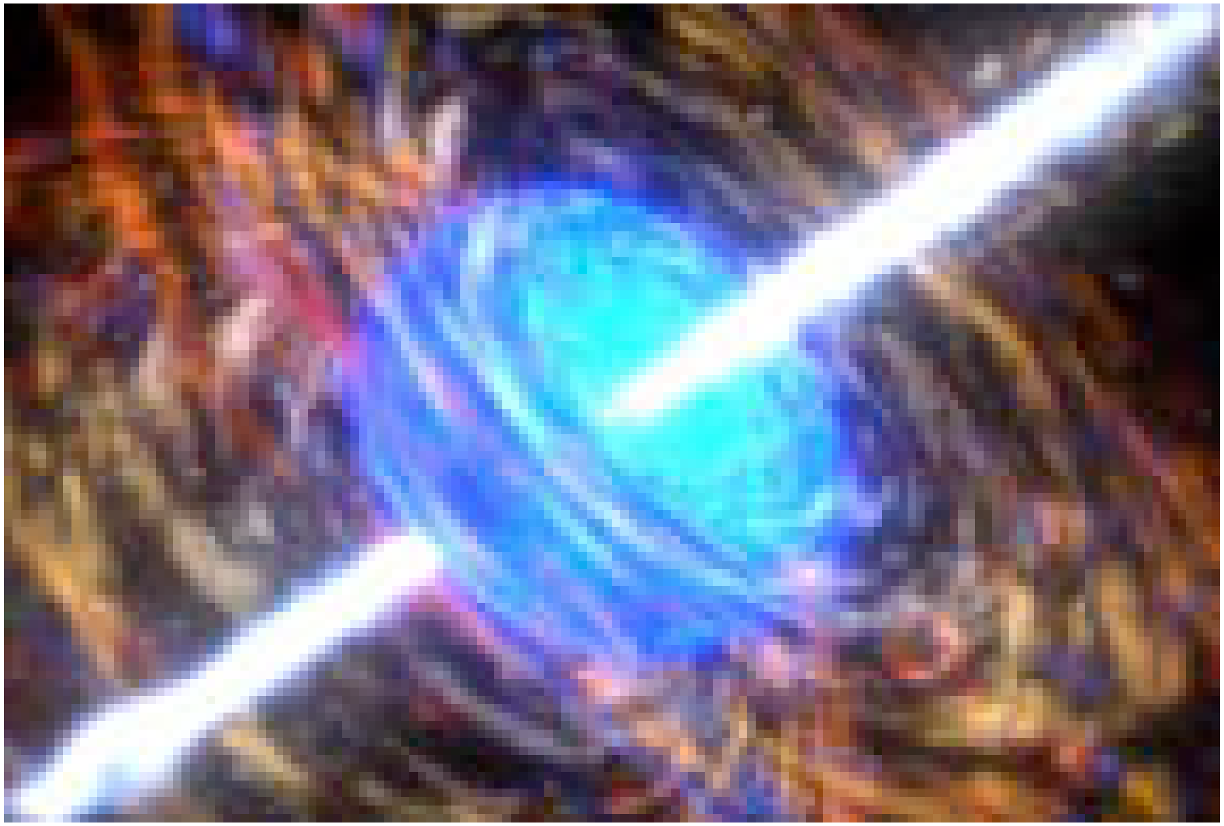


Credits: NASA/STScI/AURA

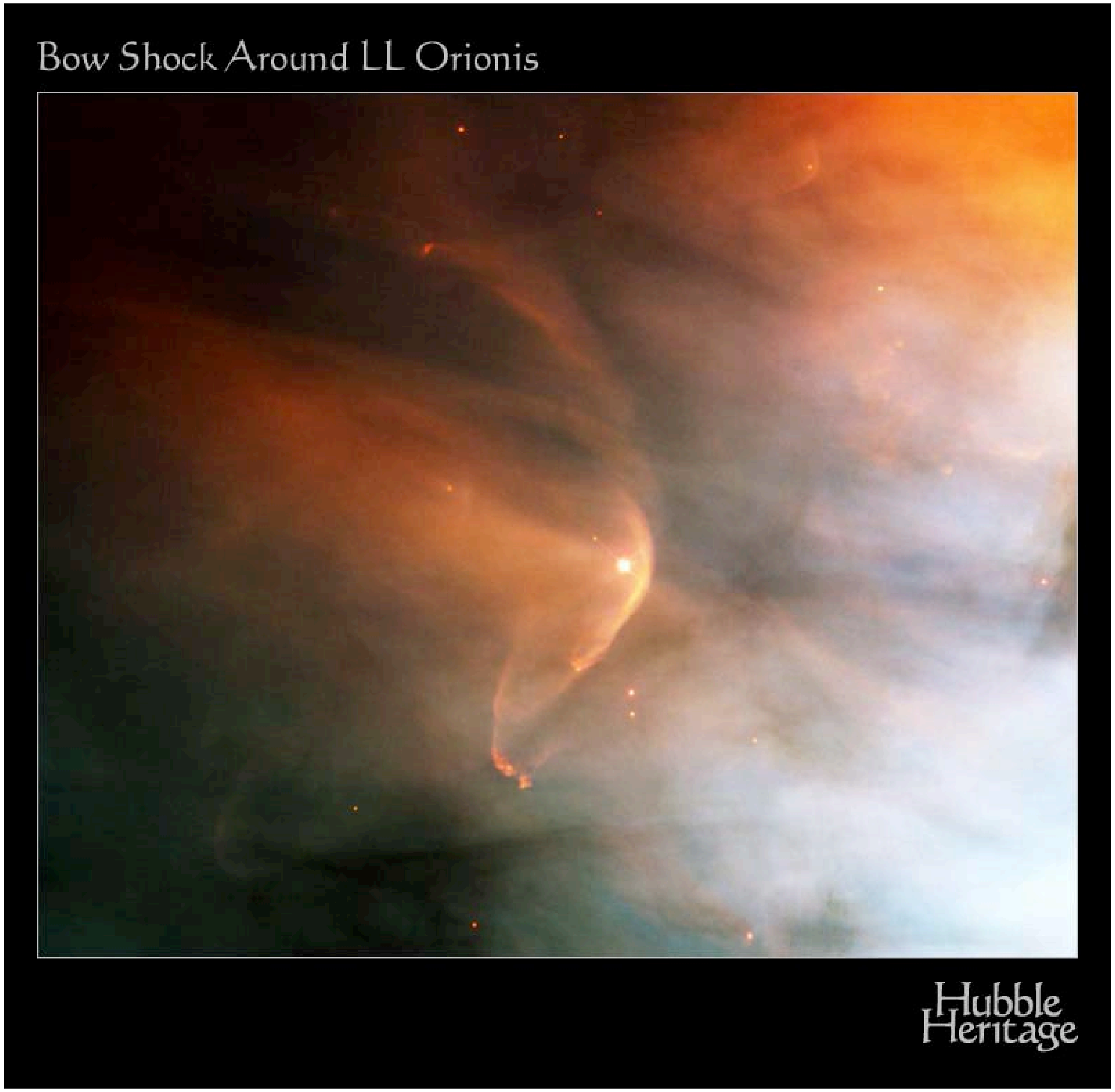
Large-Scale Galactic Fields



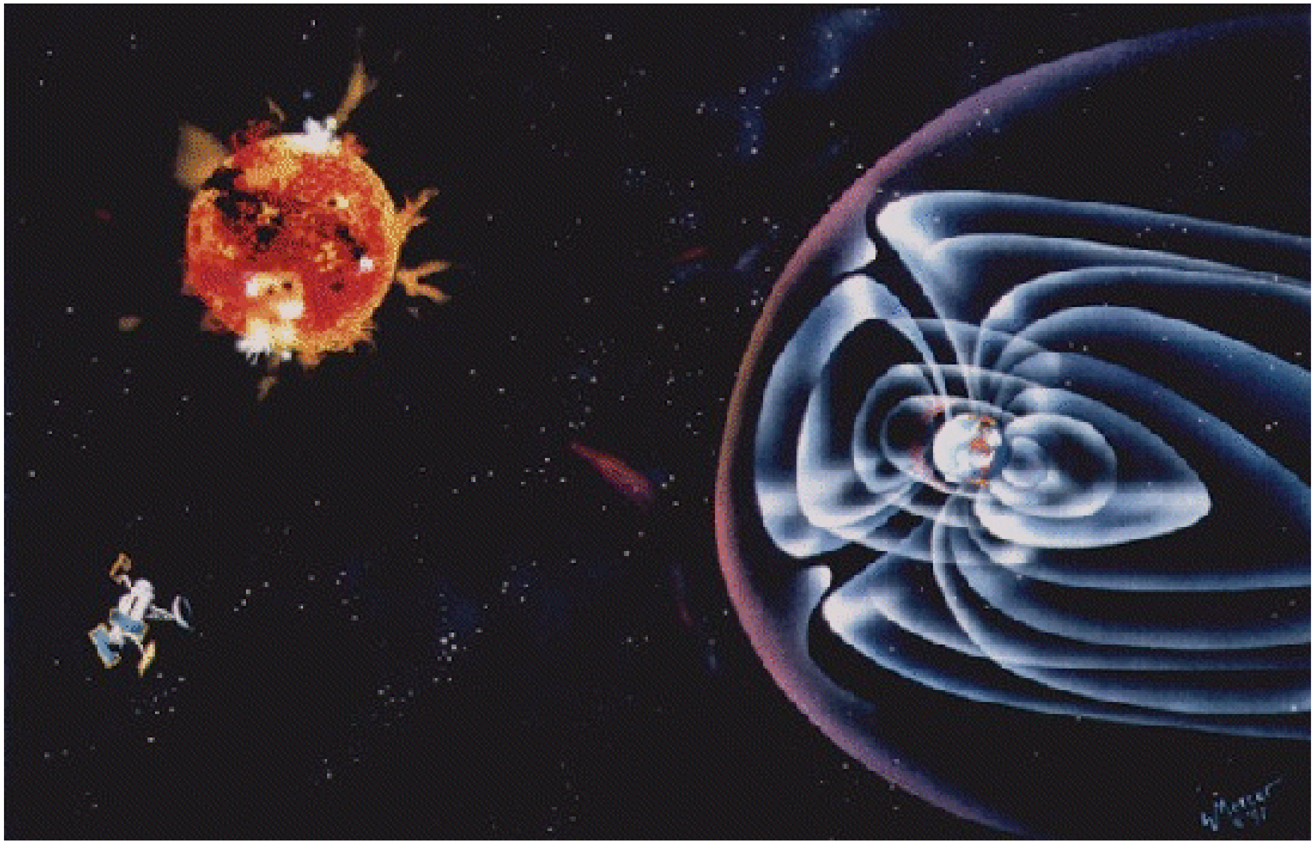
Gamma Ray Bursts



Astrospheres

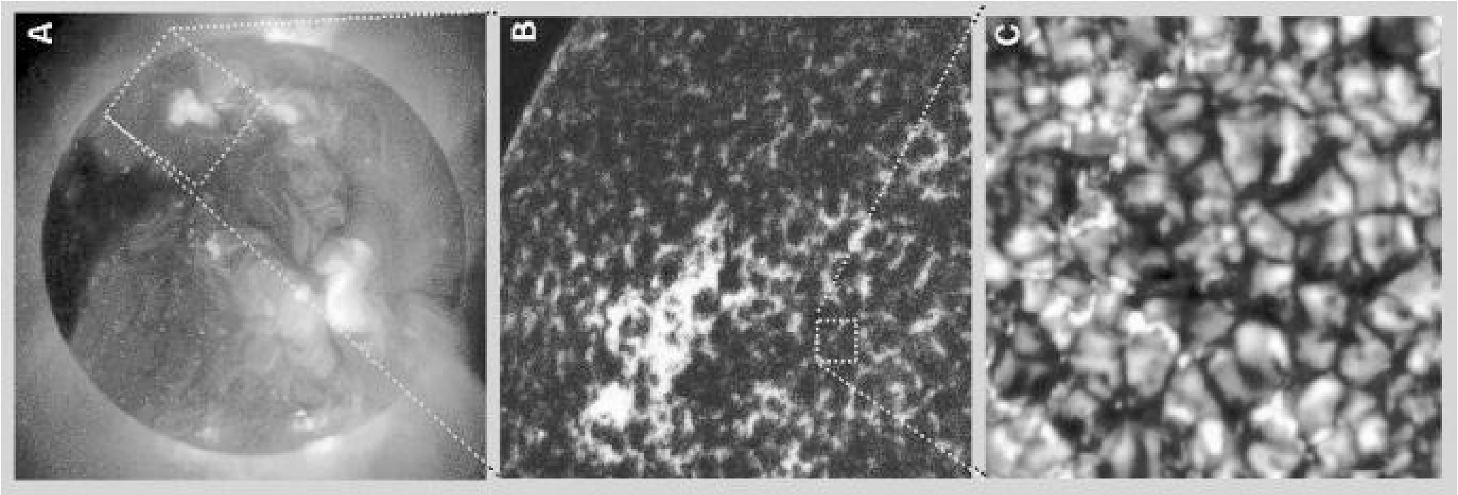


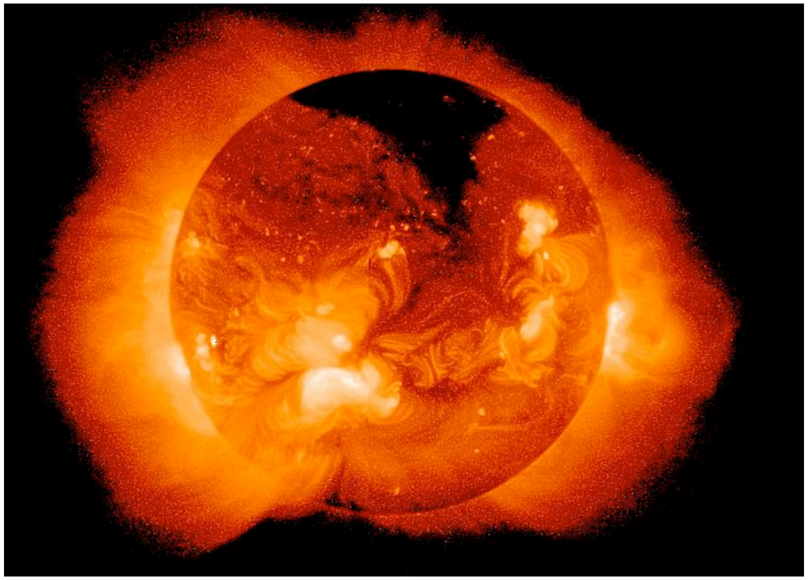
Magnetospheres of Planets

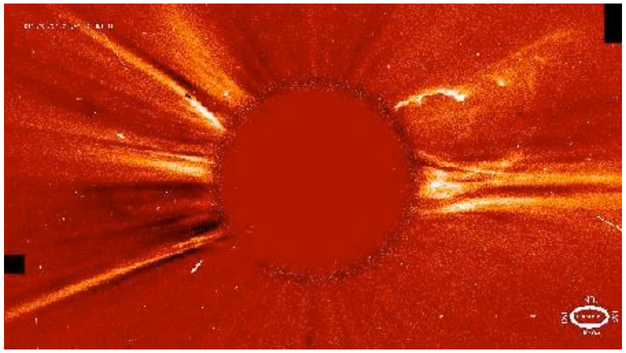


Credit: Rice

Stellar Interior and Corona

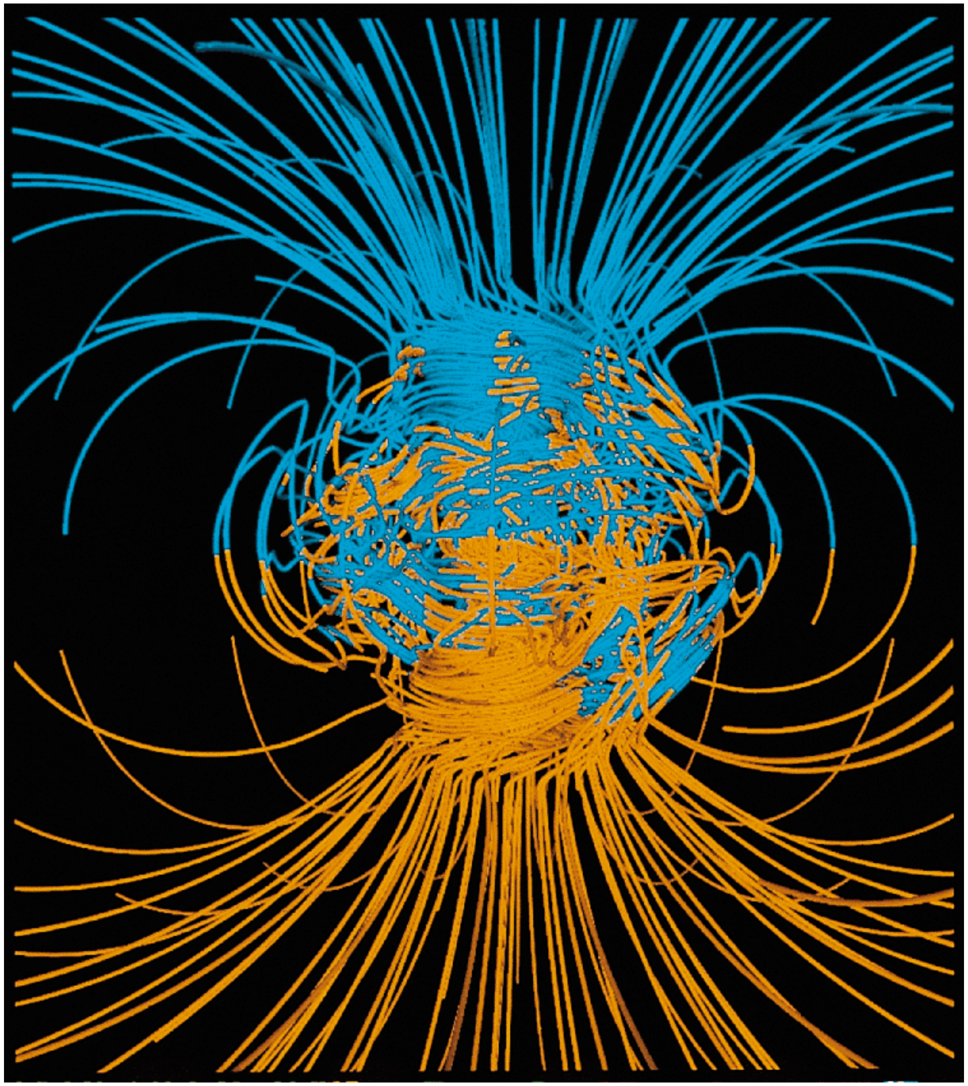






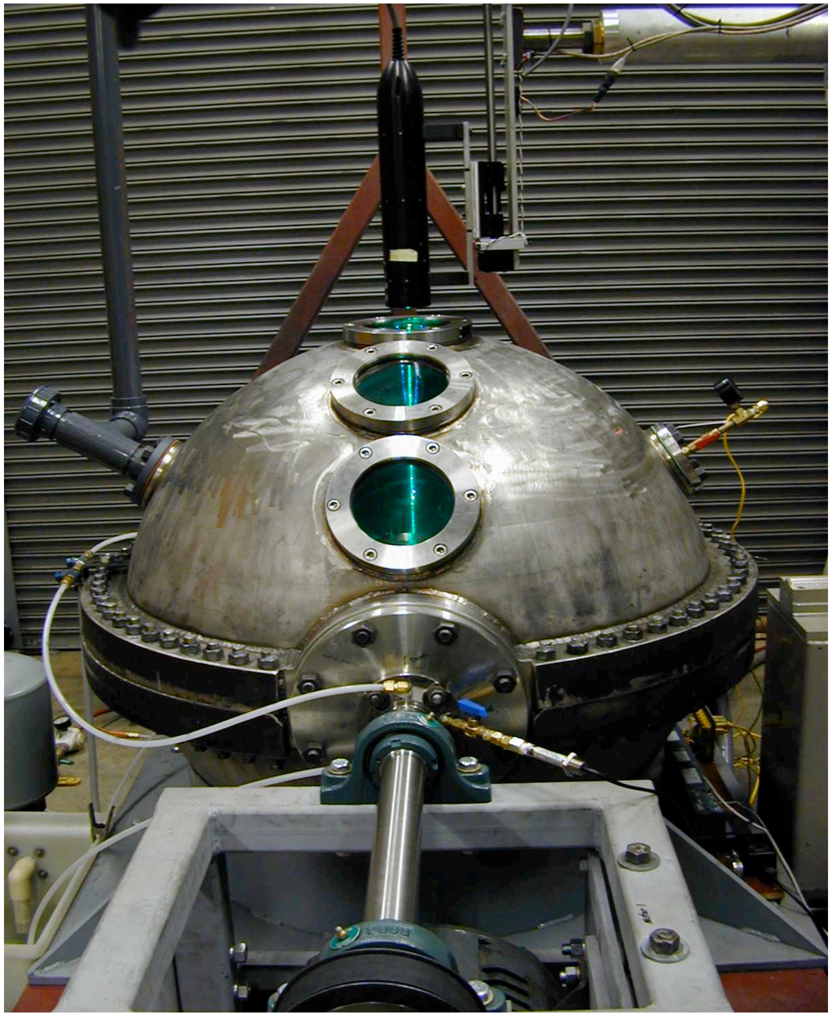
Credits: Yohkoh, SOHO and Brummel

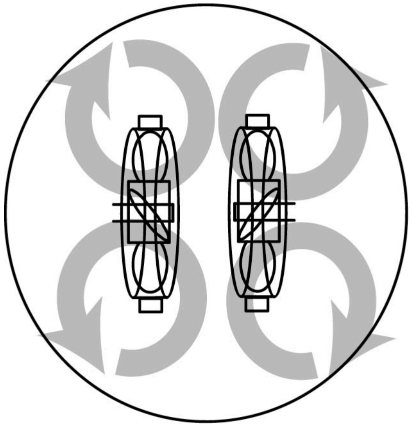
Earth Magnetic Field

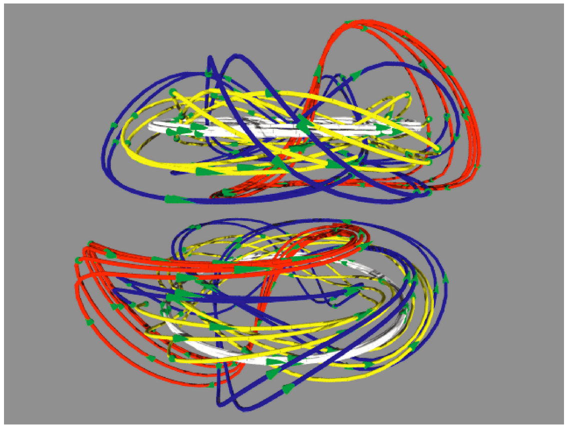


Credit: Glatzmaier

Liquid Metal Experiments

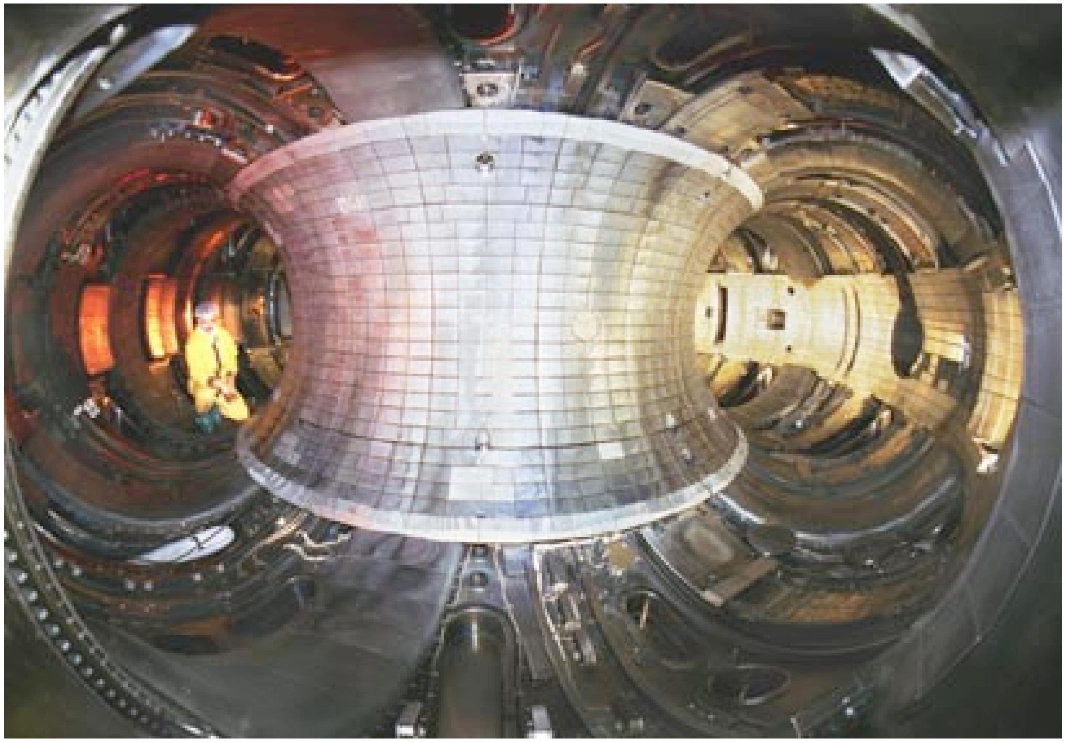


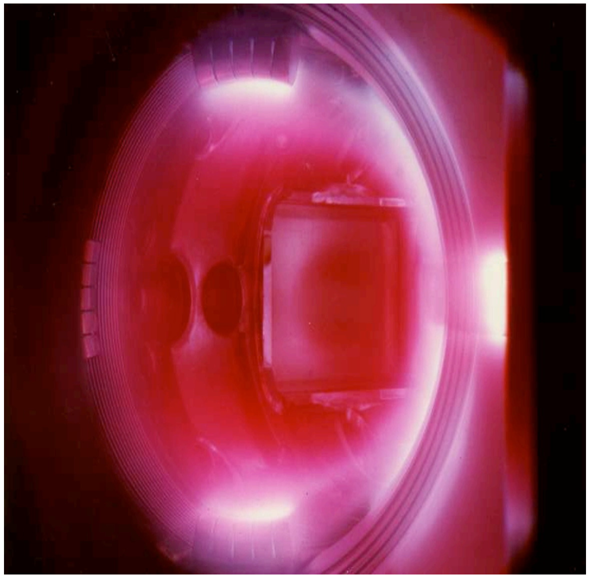




Credit: Forest (Madison Dynamo Experiment)

Tokamaks and Magnetic Confinement





Credits: PPPL/TFTR and
Forschungscentrum Jülich IPP

Magnetic Field Strengths

Intergalactic magnetic field	10^{-9} Gauss
Local interstellar cloud	1-5x10^{-6} Gauss
Galactic magnetic field	10^{-5} Gauss
Solar Wind (at 1AU)	5x10^{-5} Gauss
Interstellar molecular cloud	10^{-3} Gauss
Earth's field at ground level	1 Gauss
Solar surface field	1-5 Gauss
Massive star (pre supernova)	10^{2} Gauss
Toy refrigerator magnet	10^{2} Gauss
Sun spot field	10^{3} Gauss
Jupiter magnetic field	10^{3} Gauss
Magnetic Stars	10^{4} Gauss
Tokamak	1-10x10^{4} Gauss
White Dwarf star surfaces	10^{6} Gauss
Neutron star surface field	10^{12} Gauss
Magnetar surface field	10^{15} Gauss

MHD Approximation

The range of applications and regimes is very wide in plasma physics.

It is very important to remember which approximations/equations to use.

- Magnetohydrodynamics (MHD) equations describe flows of conducting fluids (ionized gases, liquid metals) in presence of magnetic fields.
- Lorentz forces act on charged particles and change their momentum and energy. In return, particles alter strength and topology of magnetic fields.
- MHD equations are typically derived under following assumptions:
  - Fluid approximation (often, single-fluid approximation);
  - Charge neutrality;
  - Isotropic temperature and transport coefficients;
  - No relativistic effects;
  - In ideal MHD: infinite conductivity (zero resistivity), zero viscosity and zero thermal diffusivity.

Range of validity of MHD equations, especially of ideal MHD is narrow.
Therefore, very few physical systems are truly MHD.

MHD Equations

- Strictly speaking plasma approximations should be derived from the Liouville equation. This is especially important in derivation of complex equations, e.g. two-fluid equations and Ohm's law.
- MHD equations are a highly simplified version of these equations.
- Fortunately, one can easily derive them using fluid+EM approach:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0$$

$$\rho\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\mathbf{V}\right) = \nabla p + \mathbf{f}$$

$$\frac{\partial \rho e}{\partial t} + \nabla \cdot (\rho \mathbf{V} h) = -\nabla \cdot (\mathbf{E} \times \mathbf{H})$$

$$\mathbf{f} = \rho_e \mathbf{E} + \mathbf{j} \times \mathbf{B}$$

$$\mathbf{B} = \mu \mathbf{H}$$

$$\nabla \times \mathbf{H} = \mathbf{j}$$

$$\nabla \cdot \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \cdot \mathbf{D} = \rho_e$$

Resistive MHD Ohm's law $\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{j}$

Ideal MHD Ohm's law $\mathbf{E} = -\mathbf{V} \times \mathbf{B}$

Ideal MHD Equations

$$\frac{\partial}{\partial t}\begin{pmatrix}\rho \\ \mathbf{V} \\ \rho E \\ \mathbf{B}\end{pmatrix} + \nabla\cdot\begin{pmatrix}\rho\mathbf{V} \\ \rho\mathbf{V}\mathbf{V} + (p+\frac{B^2}{2})\bar{\bar{\mathbf{I}}} - \mathbf{B}\mathbf{B} \\ \mathbf{V}(\rho E + p + \frac{B^2}{2}) - \mathbf{B}(\mathbf{V}\cdot\mathbf{B}) \\ \mathbf{V}\mathbf{B} - \mathbf{B}\mathbf{V}\end{pmatrix} = 0$$

Major Mathematical Properties:

- MHD equations form a hyperbolic system ➔ Seven families of waves (entropy, Alfvén and fast and slow magnetoacoustic waves).
- Convex space of physically admissible variables if convex EOS.
- Multiple degeneracies in the eigensystem ➔ possibility of compound waves, shock evolutionarity concerns.

Important to remember: Fluid (Euler) equations <u>are not</u> the limiting case of MHD equations in the B $\rightarrow$ 0 case in strict mathematical sense.

Resistive MHD Equations

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &+ \nabla \cdot (\rho \mathbf{v}) = 0 \\
\frac{\partial \rho \mathbf{v}}{\partial t} &+ \nabla \cdot (\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B}) + \nabla p_* = \rho \mathbf{g} + \nabla \cdot \tau \\
\frac{\partial \rho E}{\partial t} &+ \nabla \cdot (\mathbf{v}(\rho E + p_*) - \mathbf{B}(\mathbf{v} \cdot \mathbf{B})) = \rho \mathbf{g} \cdot \mathbf{v} + \nabla \cdot (\mathbf{v} \cdot \tau + \sigma \nabla T) + \nabla \cdot (\mathbf{B} \times (\eta \nabla \times \mathbf{B})) \\
\frac{\partial \mathbf{B}}{\partial t} &+ \nabla \cdot (\mathbf{v}\mathbf{B} - \mathbf{B}\mathbf{v}) = -\nabla \times (\eta \nabla \times \mathbf{B})
\end{aligned}
$$

$$
\begin{aligned}
p_* &= p + \frac{B^2}{2}, \\
E &= \frac{1}{2} v^2 + \epsilon + \frac{1}{2} \frac{B^2}{\rho}, \\
\tau &= \mu \left((\nabla \mathbf{v}) + (\nabla \mathbf{v})^{\mathrm{T}} - \frac{2}{3} (\nabla \cdot \mathbf{v}) \right)
\end{aligned}
$$

- Parabolic equations ➔ typically no need for high-resolution algorithms
- Stiff resistive, viscous, conductive time scales ➔ often need implicit algorithms

Things Can Quickly Become Complex

Plasma effects

- Reduced 2D Hall (Grasso et al, 1999)
- Electron inertia and compressibility
- 3D Hall MHD and two-fluid MHD

$$\mathbf{E}+\mathbf{V}\times\mathbf{B}=\frac{1}{S}\mathbf{J}+\frac{d_e^2}{n}\frac{d\mathbf{J}}{dt}+\frac{d_i}{n}\left(\mathbf{J}\times\mathbf{B}-\nabla\cdot\vec{p}_e\right)$$

$$\begin{aligned}
\frac{\partial F}{\partial t}+[\phi,F] &= \rho_s^2[U,\psi] \\
\frac{\partial U}{\partial t}+[\phi,U] &= [J,\psi] \\
F &= \psi+d_e^2 J \\
J &= -\nabla^2\psi
\end{aligned}$$

$$\begin{aligned}
\vec{B} &= B_0\hat{z}+\nabla\psi\times\hat{z} \\
\vec{v} &= \hat{z}\times\nabla\phi
\end{aligned}$$

Relativistic MHD

$$\frac{\partial\mathbf{W}}{\partial t}+(\nabla\cdot\mathbf{F})^{\mathrm{T}}=\mathbf{0}$$

$$\mathbf{W}=\begin{pmatrix}\Gamma_\rho \\ \Gamma^2\frac{e+p}{c^2}\mathbf{u}+\frac{1}{c^2}\mathbf{S}_{\mathrm{A}} \\ \mathbf{B} \\ \Gamma^2(e+p)-p-\Gamma\rho c^2+e_{\mathrm{A}}\end{pmatrix} \qquad \mathbf{F}=\begin{pmatrix}\Gamma\rho\mathbf{u} \\ \frac{\Gamma^2}{c^2}(e+p)\mathbf{u}\mathbf{u}+p\mathbf{I}+\mathbf{P}_{\mathrm{A}} \\ \mathbf{u}\mathbf{B}-\mathbf{B}\mathbf{u} \\ [\Gamma^2(e+p)-\Gamma\rho c^2]\mathbf{u}+\mathbf{S}_{\mathrm{A}}\end{pmatrix}^{\mathrm{T}}$$

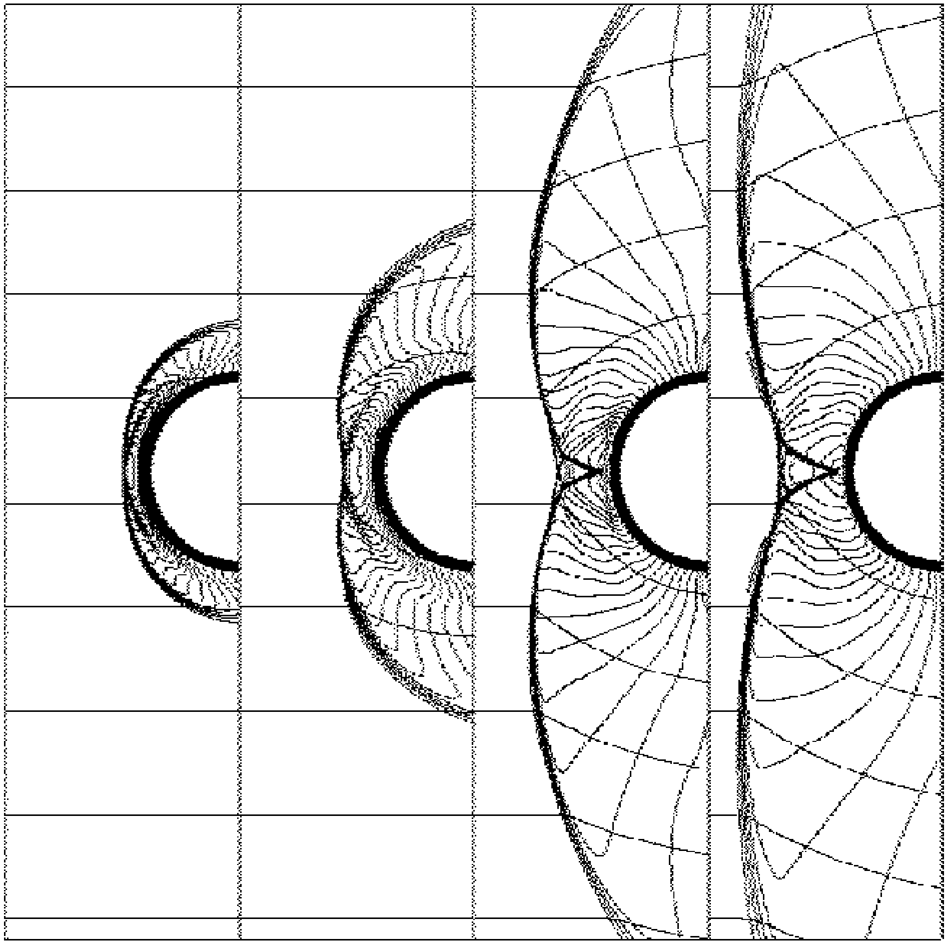
$$\Gamma=\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}, \qquad e_{\mathrm{A}}=\frac{1}{2\mu_0}\left(B^2+\frac{1}{c^2}E^2\right),$$

$$\mathbf{S}_{\mathrm{A}}=\frac{1}{\mu_0}(\mathbf{E}\times\mathbf{B}), \qquad \mathbf{P}_{\mathrm{A}}=e_{\mathrm{A}}\mathbf{I}-\frac{1}{\mu_0}\mathbf{B}\mathbf{B}-\frac{1}{\mu_0 c^2}\mathbf{E}\mathbf{E}.$$

Basic Properties of MHD Equations

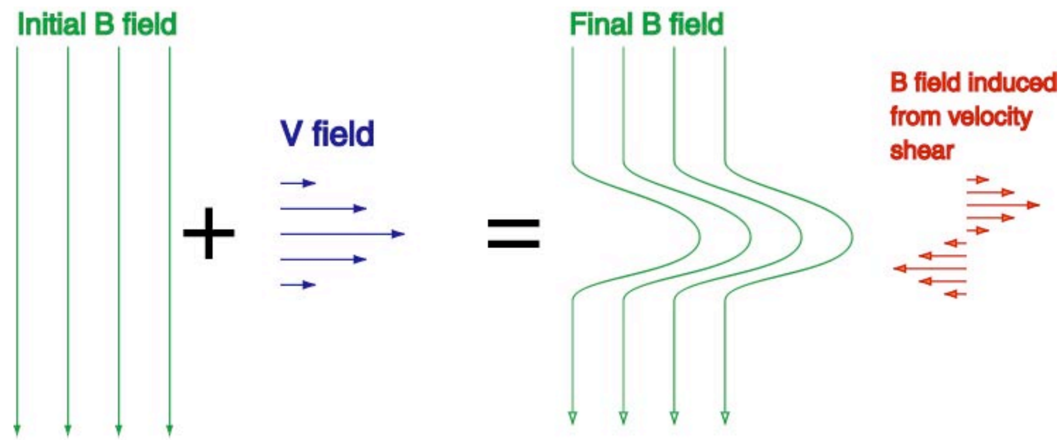
- Shock waves
  - Fast Shocks (=> acoustic shocks as B->0)
  - Slow Shocks (+Alfvén wave => tangential discontinuities as B->0)
  - Compound waves (still debates about evolutionarity)
- Pure rotational discontinuities (analogous to Alfvén waves)
- Highly non-local dissipation leading to separation of scales and explosive reconnection events
- Instabilities
  - Classic fluid instabilities (Kelvin-Helmholtz, Rayleigh-Taylor)
  - MHD instabilities (tearing mode)
  - Plasma instabilities (beam, ion-acoustic)
- Field filamentation, small-scale structures, inverse cascades
- Dynamo action

Shocks



Credit: DeSterck et al

Dynamo

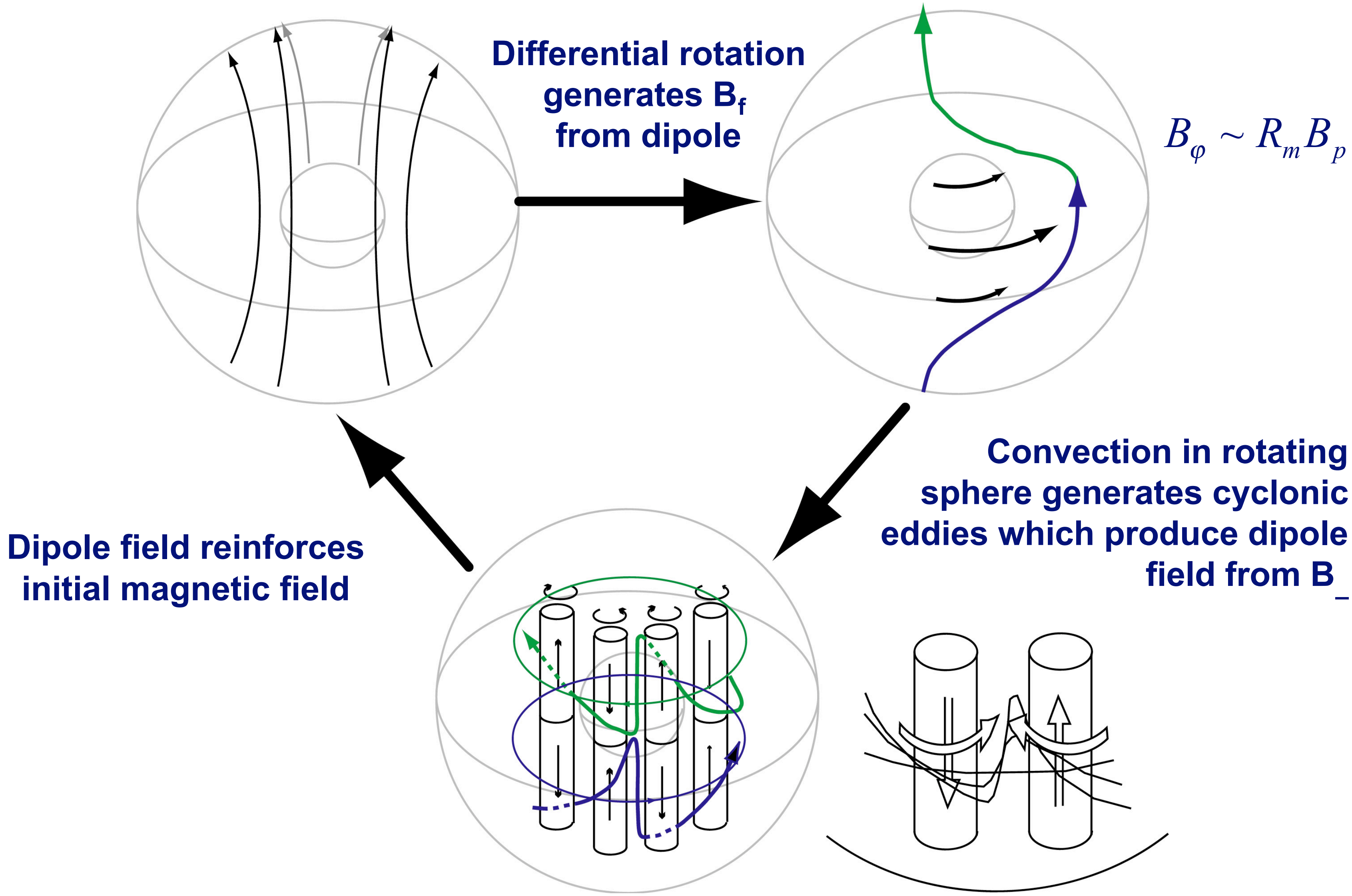


- In a fast moving, or highly conducting fluid, magnetic field lines are frozen into the moving fluid

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}$$

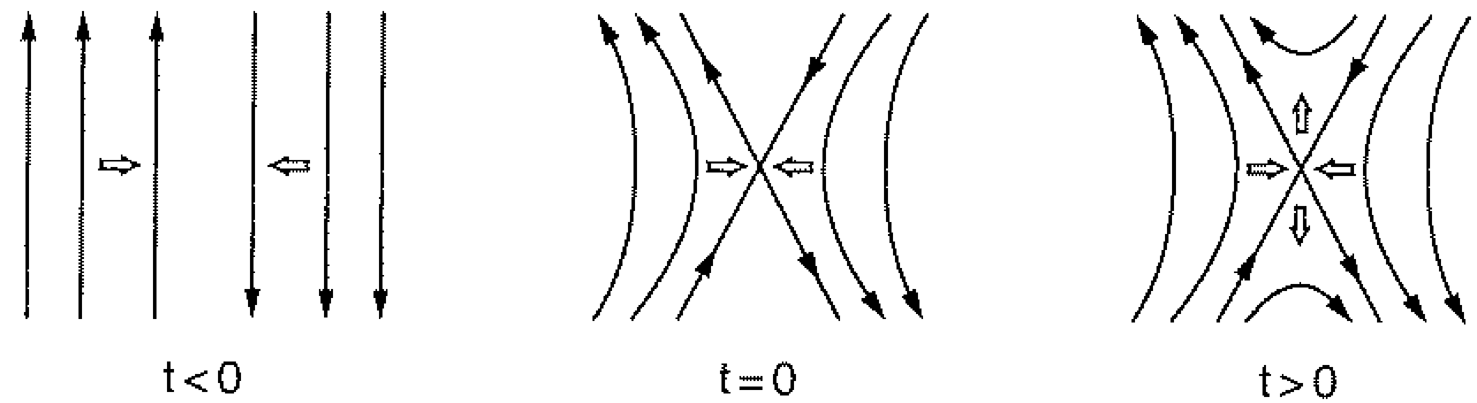
- Transverse component of field is generated and amplified
- Finite resistance leads to diffusion of field lines
- Under what conditions does dynamo occur? Where does it saturate?

Dynamo



Differential rotation
generates B_f
from dipole
$B_\varphi \sim R_m B_p$
Convection in rotating
sphere generates cyclonic
eddies which produce dipole
field from B_
Dipole field reinforces
initial magnetic field

Reconnection



- Why does reconnection occur?
- Why is it typically fast?
- What is the source of resistivity?
- How do electrons and ions behave?

Numerical Simulations

- Numerical methods should match the properties of equations.
- No need to use blindly very complex methods if solution features do not need them.
- Therefore, if flows are smooth and all physical effects are to be fully resolved, use high-order (e.g. 4^{th}) finite difference or if possible spectral codes. These codes are more accurate and easier to develop than any modern high-resolution codes.
- Use sophisticated high-resolution codes (PPM, TVD, wENO, etc.) only if solutions feature sharp discontinuities which cannot be resolved using physical diffusion operators.

High-Resolution Methods

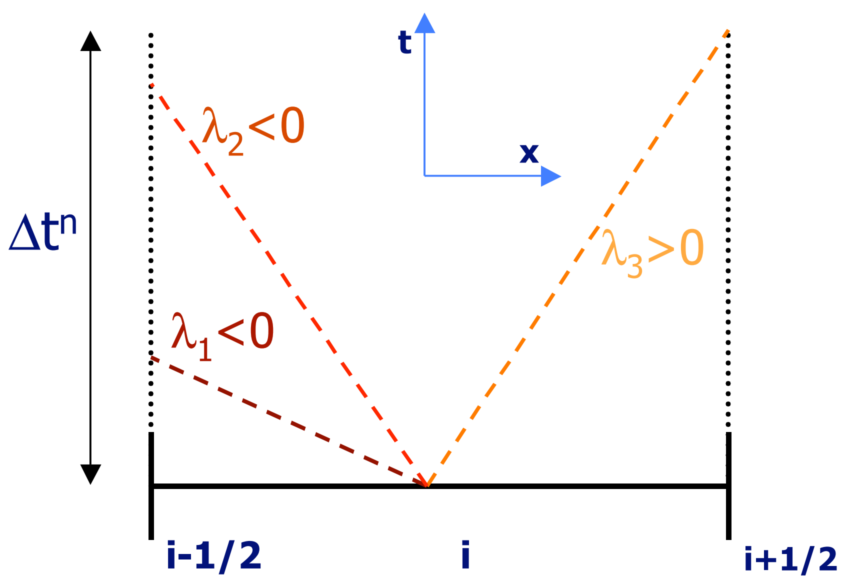
- All sophisticated high-resolution methods basically solve

$$\frac{\partial \overline{\mathrm{U}}}{\partial t} = -\frac{1}{\mathcal{V}} \iint_{\partial \mathcal{V}} \mathrm{F}_n ds + \overline{\mathrm{S}}$$

- The challenge is to evaluate source terms (typically trivial) and to compute interface fluxes (typically not trivial)
- First reconstruct solution using monotone slope limiting algorithms
- Then evaluate interface fluxes using the so-called Riemann problem. The general solution to the MHD Riemann problem is not known. Use approximate solvers (Roe, HLL*, Dai & Woodward, Balsara, Colella, …)
- Finally, give interface fluxes to a suitable time integrator end update solution. There are two time integration approaches:
  - Method of lines/ ODE/ Runge-Kutta multi-stage algorithm – simple by costly. This method is mostly used by unsplit methods.
  - One-step predictor-corrector algorithm – more difficult but faster, less costly, more robust but frequently less accurate. This method is favored by operator split methods.

One-Step Time Integration

- Consider quasi-linear form: $$\frac{\partial \boldsymbol{V}}{\partial t} = -A\frac{\partial \boldsymbol{V}}{\partial x} + \boldsymbol{S}$$
- Use Taylor expansion: $$\boldsymbol{V}^{n+\frac{1}{2}}_{i+\frac{1}{2}} = \boldsymbol{V}^{n}_{i+\frac{1}{2}} + \left(-A_i\frac{\partial \boldsymbol{V}_i}{\partial x} + \boldsymbol{S}_i\right)\frac{\Delta t^n}{2}$$
- Characteristic tracing: $$A = R\Big(\Lambda_{(>0)} + \Lambda_{(<0)}\Big)L$$



$$\hat{\boldsymbol{V}}^{n+\frac{1}{2}}_{i+\frac{1}{2},L} = \boldsymbol{V}^{+}_{i+\frac{1}{2},L} - \sum_{\lambda_i^{\#}>0}\left[\boldsymbol{l}_i^{\#}\cdot\left(\boldsymbol{V}^{+}_{i+\frac{1}{2},L} - \boldsymbol{V}^{\#}_{i+\frac{1}{2},L}\right)\right]\boldsymbol{r}_i^{\#} + \frac{\Delta t}{2}\boldsymbol{S}_i,$$

$$\hat{\boldsymbol{V}}^{n+\frac{1}{2}}_{i-\frac{1}{2},R} = \boldsymbol{V}^{-}_{i-\frac{1}{2},R} - \sum_{\lambda_i^{\#}<0}\left[\boldsymbol{l}_i^{\#}\cdot\left(\boldsymbol{V}^{-}_{i-\frac{1}{2},R} - \boldsymbol{V}^{\#}_{i-\frac{1}{2},R}\right)\right]\boldsymbol{r}_i^{\#} + \frac{\Delta t}{2}\boldsymbol{S}_i\,.$$

Characteristic Matrix

$$\frac{\partial \mathrm{W}}{\partial t} + A_x \frac{\partial \mathrm{W}}{\partial x} + A_y \frac{\partial \mathrm{W}}{\partial y} + A_z \frac{\partial \mathrm{W}}{\partial z} = 0$$

$$\mathrm{W} = (\rho \; u \; v \; w \; p \; B_x \; B_y \; B_z)^{\mathrm{T}}$$

$$\mathrm{A}_x = \begin{bmatrix} u & \rho & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & u & 0 & 0 & \frac{1}{\rho} & 0 & \frac{B_y}{\rho} & \frac{B_z}{\rho} \\ 0 & 0 & u & 0 & 0 & 0 & -\frac{B_x}{\rho} & 0 \\ 0 & 0 & 0 & u & 0 & 0 & 0 & -\frac{B_x}{\rho} \\ 0 & \gamma p & 0 & 0 & u & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & u & 0 & 0 \\ 0 & B_y & -B_x & 0 & 0 & 0 & u & 0 \\ 0 & B_z & 0 & -B_x & 0 & 0 & 0 & u \end{bmatrix}$$

Characteristic Speeds and Eigenvectors

$$\lambda_e = u_n, \qquad a = \sqrt{\frac{\gamma p}{\rho}}$$

Magnetoacoustic

$$\lambda_a^{\pm} = u_n \pm |V_{An}|, \qquad \mathbf{V}_A = \frac{\mathbf{B}}{\sqrt{\rho}}$$

Alfvén

$$\lambda_{f,s}^{\pm} = u_n \pm c_{f,s}, \qquad c_{f,s}^2 = \frac{1}{2}\left(a^2 + V_A^2 \pm \sqrt{(a^2+V_A^2)^2 - 4a^2V_{An}^2}\,\right)$$

$$\begin{aligned}
\lambda_e:\ \mathbf{l}_e &= \left(1,\ 0,\ 0,\ 0,\ -\frac{1}{a^2},\ 0\ ,0\ ,0\right),\\
\mathbf{r}_e &= (1,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0)^{\mathbf{T}};\\
\lambda_a^{\pm}:\ \mathbf{l}_a^{\pm} &= \left(0,\ 0,\ -B_{\tau_2}\sigma_{B_n},\ B_{\tau_1}\sigma_{B_n},\ 0,\ 0, \pm\frac{B_{\tau_2}}{\sqrt{\rho}}, \mp\frac{B_{\tau_1}}{\sqrt{\rho}}\right),\\
\mathbf{r}_a^{\pm} &= \left(0,\ 0,\ -B_{\tau_2}\sigma_{B_n},\ B_{\tau_1}\sigma_{B_n},\ 0,\ 0, \pm B_{\tau_2}\sqrt{\rho}, \mp B_{\tau_1}\sqrt{\rho}\right)^{\mathbf{T}};\\
\lambda_{f,s}^{\pm}:\ \mathbf{l}_{f,s}^{\pm} &= \left(0,\ \pm\rho c_{f,s},\ \mp\frac{\rho c_{f,s}B_nB_{\tau_1}}{\rho c_{f,s}^2 - B_n^2},\ \mp\frac{\rho c_{f,s}B_nB_{\tau_2}}{\rho c_{f,s}^2 - B_n^2},\ 1,\right.\\
&\qquad\qquad \left. 0,\ \frac{\rho c_{f,s}B_{\tau_1}}{\rho c_{f,s}^2 - B_n^2},\ \frac{\rho c_{f,s}B_{\tau_2}}{\rho c_{f,s}^2 - B_n^2}\right),\\
\mathbf{r}_{f,s}^{\pm} &= \left(\rho,\ \pm c_{f,s},\ \mp\frac{c_{f,s}B_nB_{\tau_1}}{\rho c_{f,s}^2 - B_n^2},\ \mp\frac{c_{f,s}B_nB_{\tau_2}}{\rho c_{f,s}^2 - B_n^2},\ \gamma p,\right.\\
&\qquad\qquad \left. 0,\ \frac{\rho c_{f,s}B_{\tau_1}}{\rho c_{f,s}^2 - B_n^2},\ \frac{\rho c_{f,s}B_{\tau_2}}{\rho c_{f,s}^2 - B_n^2}\right)^{\mathbf{T}};\\
\lambda_d:\ \mathbf{l}_d &= (0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0),\\
\mathbf{r}_d &= (0,\ 0,\ 0,\ 0,\ 0,\ 1,\ 0,\ 0)^{\mathbf{T}}.
\end{aligned}$$

Specific MHD Issues

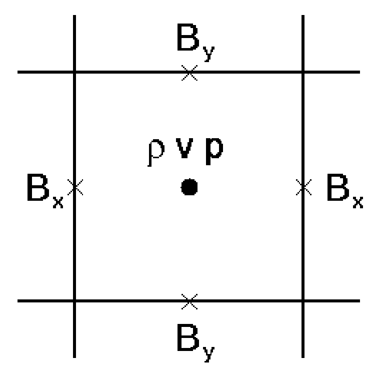
Most algorithms developed for fluid equations work for MHD.
However, $\nabla \cdot \mathbf{B} = 0$ is a cause of perpetual concern. Methods in use:

- Projection (Brackbill and Barnes, 1980):
$$\nabla^2 \varphi = \nabla \cdot \mathbf{B}$$
$$\mathbf{B} <= \mathbf{B} - \nabla \cdot \varphi$$

- Advection (Powell et al, 1999, Dellar, 2001, Dedner et al, 2002) and diffusion (Marder, 1987):
$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{VB} - \mathbf{BV}) = -\mathbf{V}(\nabla \cdot \mathbf{B}) + \eta \nabla (\nabla \cdot \mathbf{B})$$

- Constrained transport (Evans and Hawley, 1988; Stone and Norman, 1992; Dai and Woodward, Balsara, Balsara and Spicer, Tóth, 90s-present):

  Advance volumetric variables using Gauss's
  and surface variables using Stokes; theorems.



More Subtle Issues

- Magnetic field reconstruction on AMR grids
  - Preserving $\nabla \cdot \mathbf{B} = 0$ is not guaranteed on adaptive grids even if method preserves it to round-off precision
  - Ignore this issue and have the method deal with it if it can
  - Use solenoidality preserving quadratic interpolation (Balsara, Tóth and Roe) to construct prolongation and restriction operators
- Stiffness due to high Alfvén speed
  - Alfvén speed goes to the speed of light if B is strong and ρ is small
  - Previous derivation ignored displacement currents, however true Ampère's law is

$$\mathbf{j} = \nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t}$$

  - Then momentum equation becomes

$$\frac{\partial}{\partial t}\left(\rho \mathbf{V} + \frac{1}{c^2}\mathbf{S}\right) + \nabla \cdot \left(\rho \mathbf{V}\mathbf{V} + p\mathbf{I} + \left(\frac{B^2}{2}\mathbf{I} - \mathbf{B}\mathbf{B} - \frac{1}{c^2}\mathbf{E}\mathbf{E}\right)\right) = 0$$

  - This limits Alfvén speed to the speed of light
  - Often can reset the speed of light to smaller values (Boris correction)

How Can One Do This?

- Write your own code
  - Time consuming and difficult
  - Priceless experience if you plan to do serious computational physics
  - No book will teach you what you will learn in one year of own work

- Use available codes (FLASH, BATS-R-US, AMRCLAW, BEARCLAW, Enzo, ZEUS, CHOMBO, NIRVANA, NIMROD, SAMRAI, VAC, M3D, lab codes, in-house codes, …)
  - Learning curves are steep but users typically get started quickly
  - Immediately gain access to years of (someone else's) expertise
  - Will have to completely trust it and often will have no clue about what to do and who to blame

FLASH – Extensible Application Code

- Compressible hydro and MHD (PPM, TVD), implicit incompressible hydro
- Special relativistic hydro and MHD (PPM, TVD)
- Hall MHD (reduced in 2D, full in 3D)
- Equations of state:
  - Partially degenerate stellar EOS (with Coulomb corrections)
  - Non-degenerate EOS for solar photospheric conditions
  - Mixture of perfect gases
- Source terms:
  - Nuclear burning – variety of reaction networks
  - Radiative cooling (RS, MEKAL, optically thin plasma)
  - Equilibrium and non-equilibrium ionization
- Gravitational field:
  - Externally imposed (constant, plane-parallel, point source)
  - Self gravity: multipole, multigrid, FFT
- Diffusive transport processes:
  - Viscous, thermal, resistive
- General-purpose solvers and algorithms (FFT, multigrid, linalg, particles, …)
- Arbitrary geometry

MHD Module in FLASH

Current:

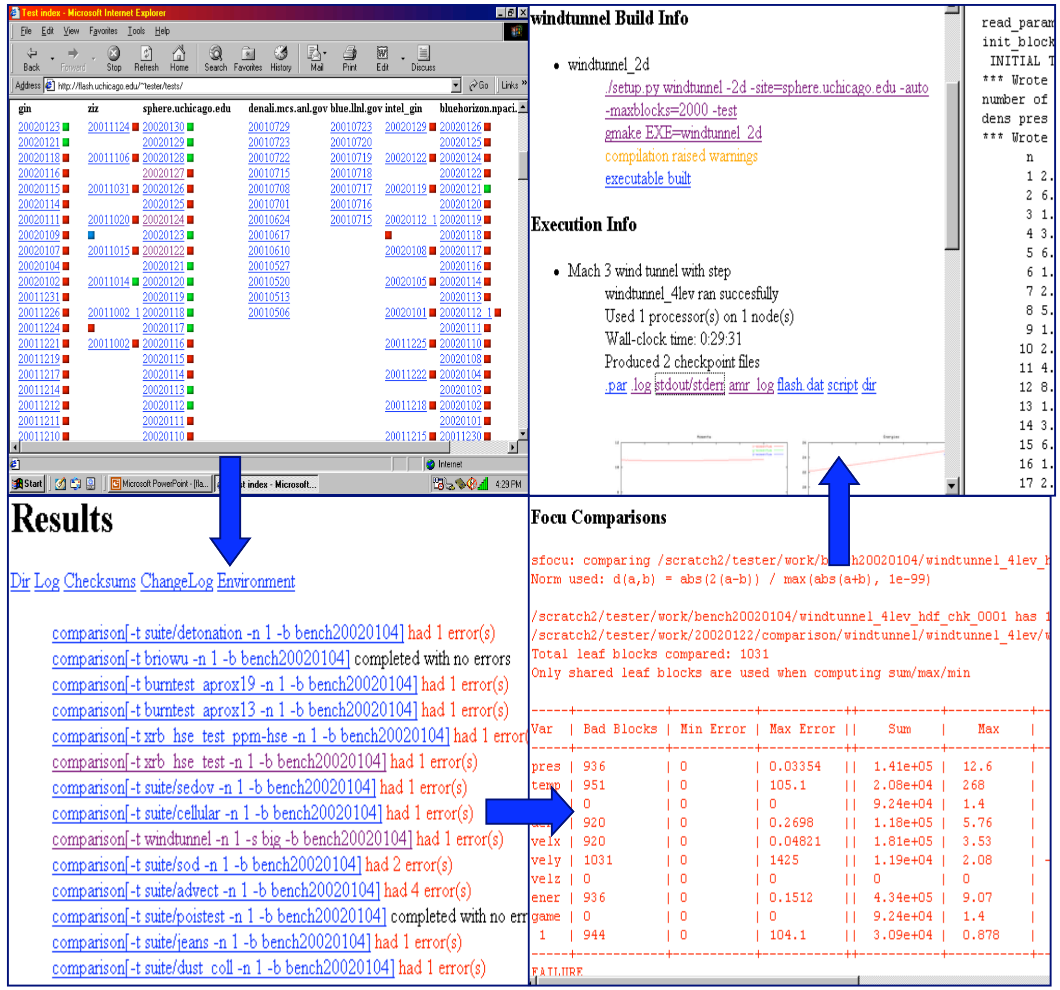
- Compressible (ideal and resistive) MHD equations
- Advective terms are discretized using slope-limited TVD scheme with full characteristic decomposition
- Diffusive terms are discretized using central finite differences.
- Choice of flux functions in the latest version (accuracy vs. robustness)
- Explicit, Hancock-type characteristic time integrator
- Multiple species using a Lagrangian algorithm
- Variable transport coefficients and general equations of state (Vinokur and Montagné, 1990; Colella and Glaz, 1985)
- Advection+diffusion (default) and projection methods to kill monopoles
- Coupling to FLASH code source terms and self-gravity modules
- Interoperability with FLASH hydro module interfaces at evolve level
- 2D reduced (Grasso et al, 1999) and 3D (Huba and Rudakov, 2002) Hall MHD equations (CMRS)

Work in progress:

- Special relativistic (1st version implemented) and semi-relativistic MHD
- Arbitrary geometry (cylindrical geometry already implemented)
- Implicit algorithms

General Features of FLASH

- Not a framework or PSE, but extensible, modular application code
- Emphasis on:
  - Performance
  - Scalability
  - Portability
  - Testing
  - Usability
- External libraries:
  - MPI (Parallelization)
  - Paramesh 2/3(AMR)
  - HDF4/HDF5 (I/O)
  - IDL/PVTK (Viz)
  - PAPI, hypre, pfft
- 600,000 lines
  in F90/C/Python/sh



Advantages of FLASH Approach

- Some advantages of FLASH
  - tested nightly
  - constantly ported to new platforms
  - i/o optimizied independently
  - visualization developed independently
  - documentation manager
  - user support
  - bug database
  - performance measured regularly
  - AMR (tested/documented independently)
  - coding standards enforcement scripts
  - debugged frequently (lint, forcheck)
  - sophisticated versioning, repository management
  - possible interplay with other physics modules (particles, etc.)
  - mechanisms to incorporate external contributions and modules

Software Process

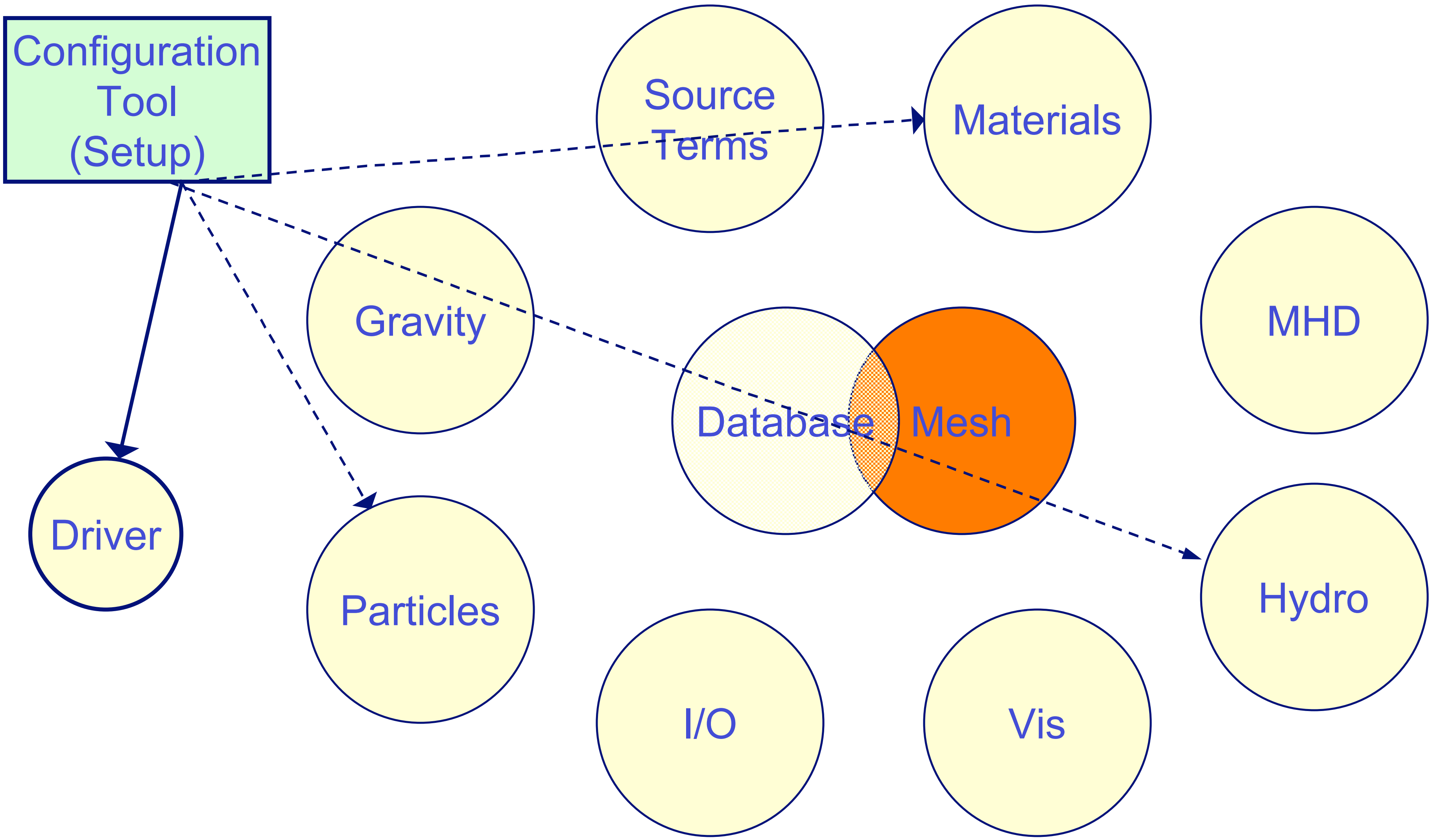
- Three levels of interaction with FLASH
  A. End-users
    - Run an existing problem

  B. Module/problem contributors
    - Use database Module interface but unaware of FLASH internals

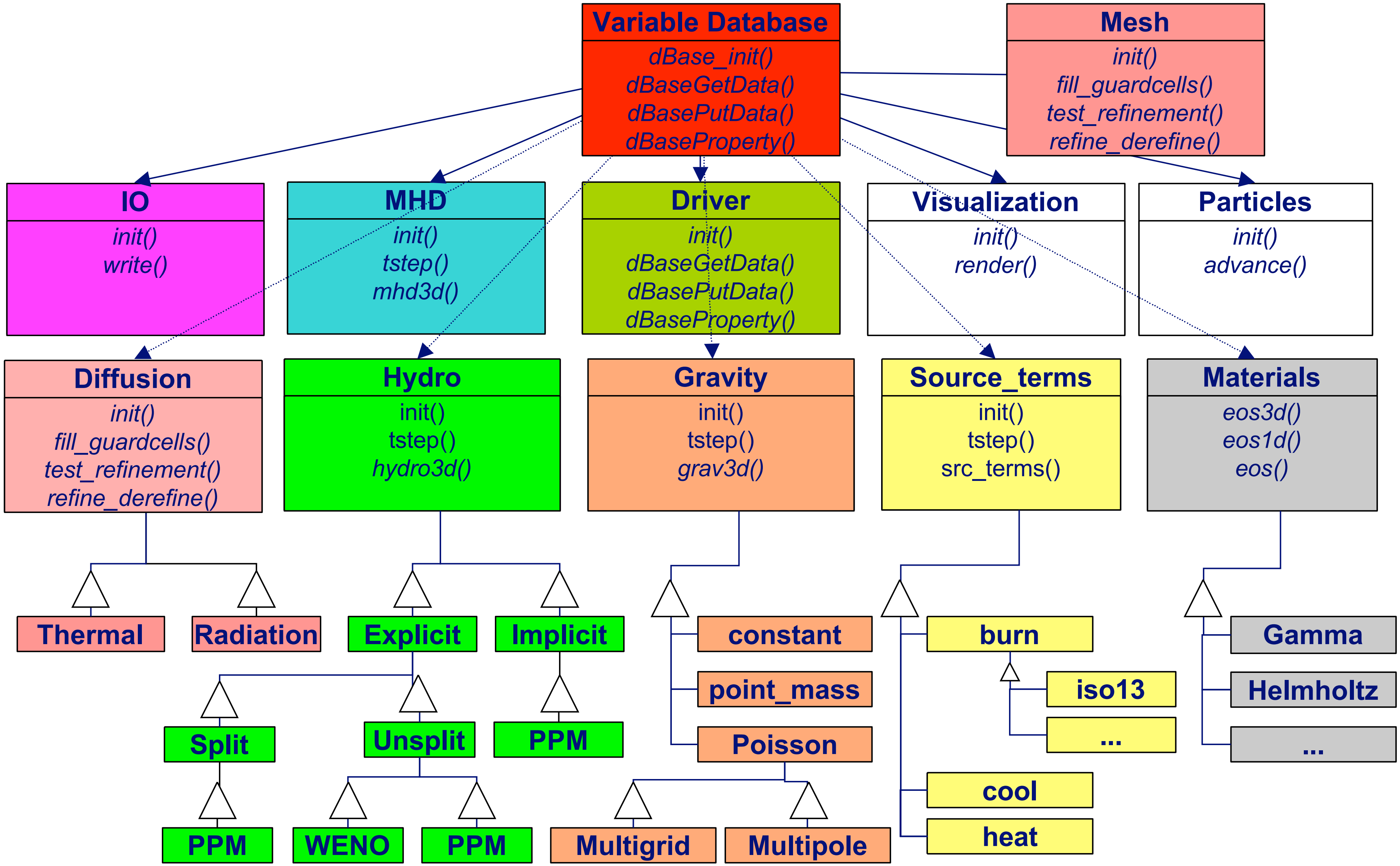
  C. FLASH developers
    - Work on general framework issues, utility modules, performance, portability, etc. according to needs of astrophysicists.

- Ultimate vision -- mature process
  - Physicists lean toward A and B
  - Programmers/software engineers lean toward C
  - Computer scientists can be involved at any level
  - Everybody contributes to design process. Software architect must make final decisions on how to implement plan
  - Everyone does what he/she likes and needs to do the most

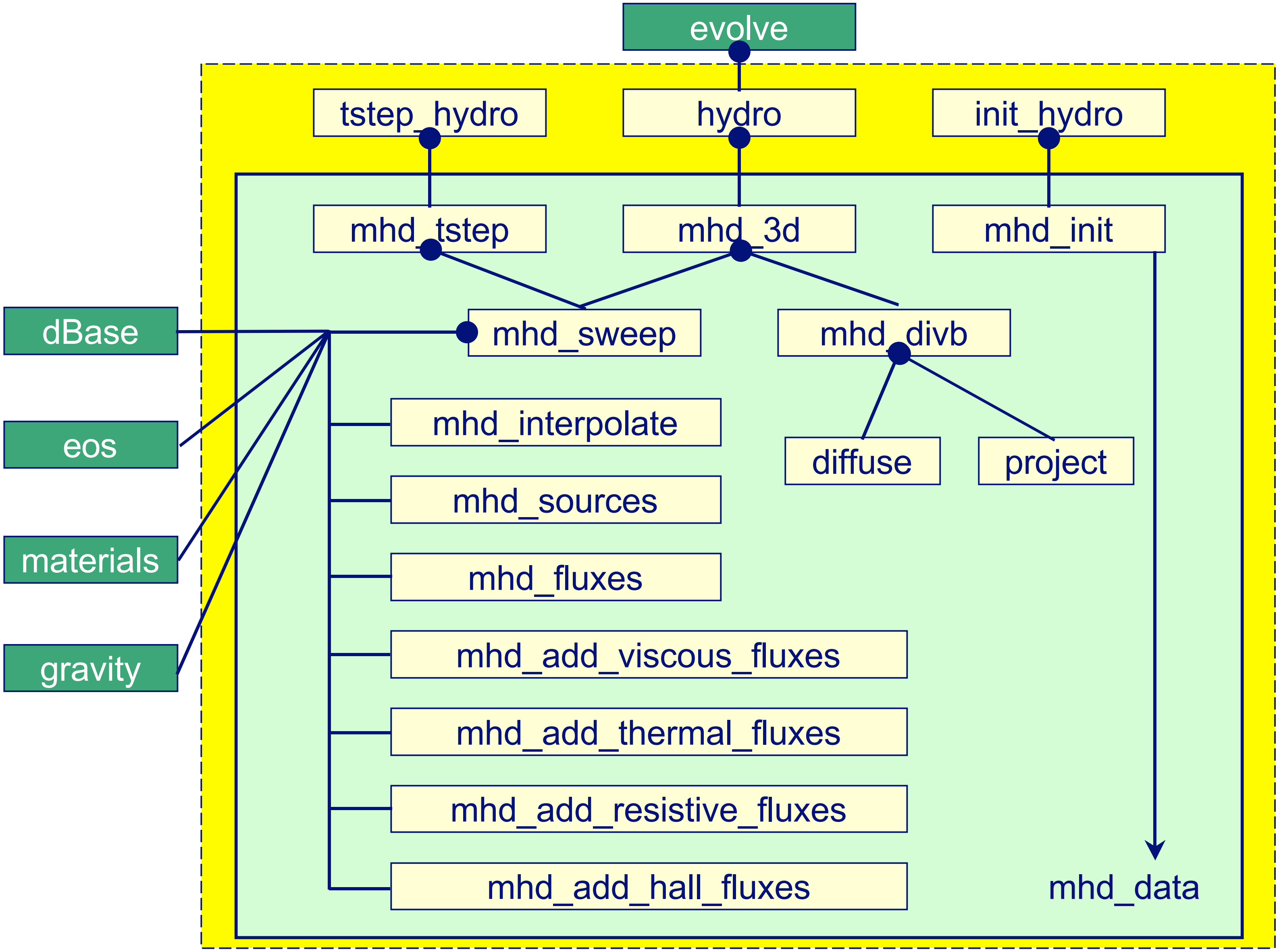
Setup Tool: Building an Application



Structure of FLASH Modules



MHD Module (source/hydro/mhd) Structure



MHD Config

```
# Required Modules

REQUIRES driver
REQUIRES materials/eos
REQUIRES materials/viscosity
REQUIRES materials/conductivity
REQUIRES materials/magnetic_resistivity

DEFAULT   divb_diffuse
EXCLUSIVE divb_diffuse divb_project

# Required Variables

VARIABLE dens   ADVECT NORENORM   CONSERVE # density
VARIABLE velx   ADVECT NORENORM NOCONSERVE # x-velocity
VARIABLE vely   ADVECT NORENORM NOCONSERVE # y-velocity
VARIABLE velz   ADVECT NORENORM NOCONSERVE # z-velocity
VARIABLE pres   ADVECT NORENORM NOCONSERVE # pressure
VARIABLE ener   ADVECT NORENORM NOCONSERVE # specific total energy
VARIABLE gamc NOADVECT NORENORM NOCONSERVE # sound-speed gamma
VARIABLE magx   ADVECT NORENORM   CONSERVE # x-magnetic field
VARIABLE magy   ADVECT NORENORM   CONSERVE # y-magnetic field
VARIABLE magz   ADVECT NORENORM   CONSERVE # z-magnetic field
VARIABLE divb NOADVECT NORENORM NOCONSERVE # divergence of B
VARIABLE temp NOADVECT NORENORM NOCONSERVE # temperature
VARIABLE eint NOADVECT NORENORM NOCONSERVE # specific internal energy

GUARDCELLS 2

# MHD Parameters

PARAMETER cfl           REAL      1.0    # CFL condition
PARAMETER UnitSystem    STRING   "none"  # Unit system (SI/cgs/none)
PARAMETER killdivb      BOOLEAN   TRUE   # Enable/disable DivB cleaning
PARAMETER resistive_mhd BOOLEAN   FALSE  # Turn on/off resistive terms
```

How to Setup a New Problem

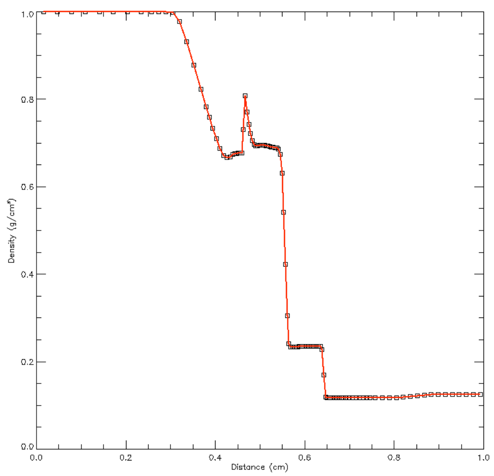
- Create Config and flash.par files

```
# Configuration file for MHD Rayleigh-Taylor problem

REQUIRES driver/time_dep
REQUIRES hydro/mhd
REQUIRES materials/eos/gamma
REQUIRES gravity/constant

NUMSPECIES 2

# Problem specific parameters

PARAMETER rho_heavy    REAL      2.0    # Density of heavy fluid
PARAMETER rho_light    REAL      1.0    # Density of light fluid

PARAMETER Bx0          REAL      0.0    # Initial Bx component
PARAMETER By0          REAL      0.0    # Initial By component
PARAMETER Bz0          REAL      0.0    # Initial Bz component
```

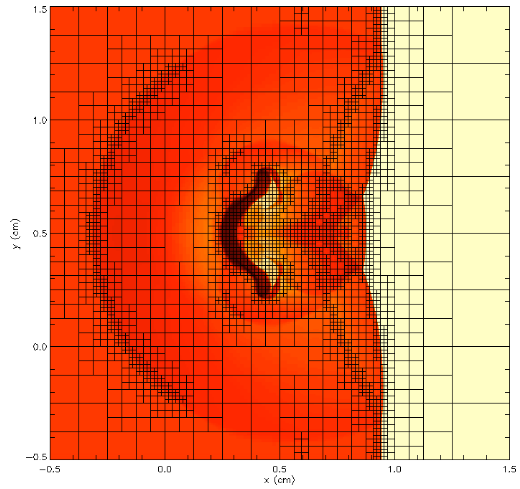
- Create init_block.F90 exactly as you would do for hydro. Just do not forget to set magnetic field variables in the initialization routine.
- Do not add magnetic pressure to total specific energy, because Flash EOS routines assume a specific expression for it.
- May need to write custom boundary conditions in user_bnd.F90, because built-in boundary conditions in FLASH assume hydro case.
- Write custom functions and do not forget to add them to Makefile.

Verification Tests

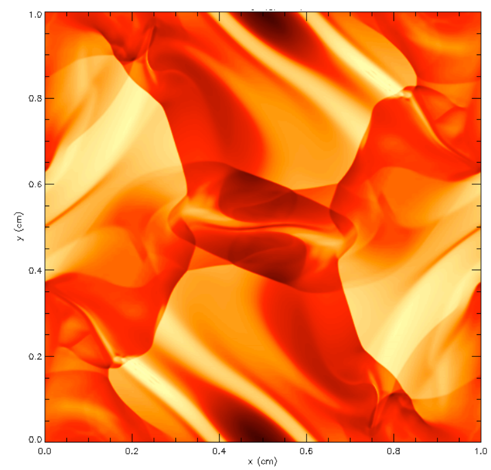
Brio-Wu



MHD Shock-Cloud Interaction

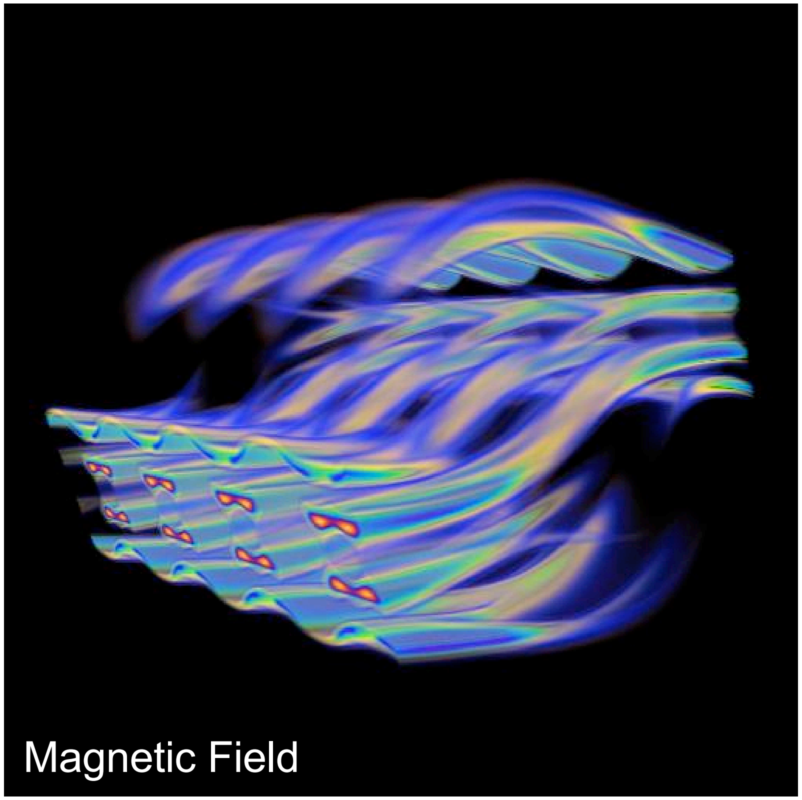


Orszag-Tang

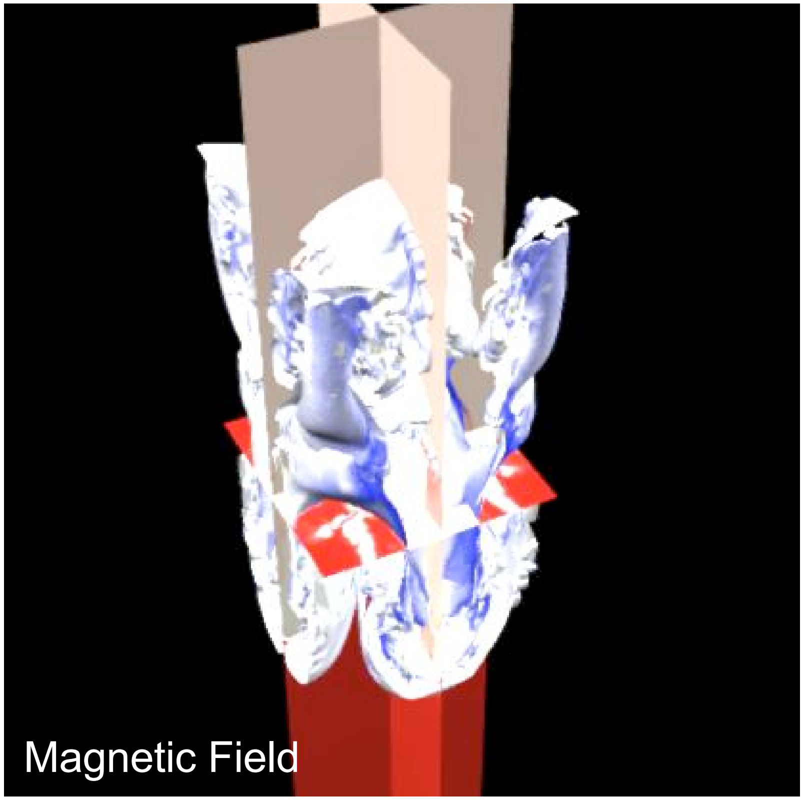


Fundamental Instabilities

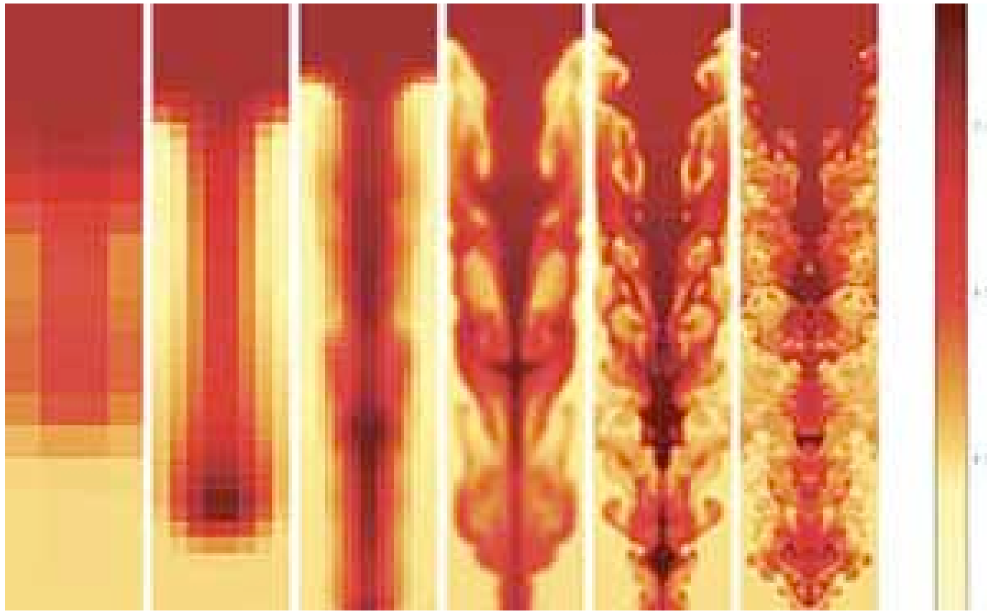
Kelvin-Helmholtz

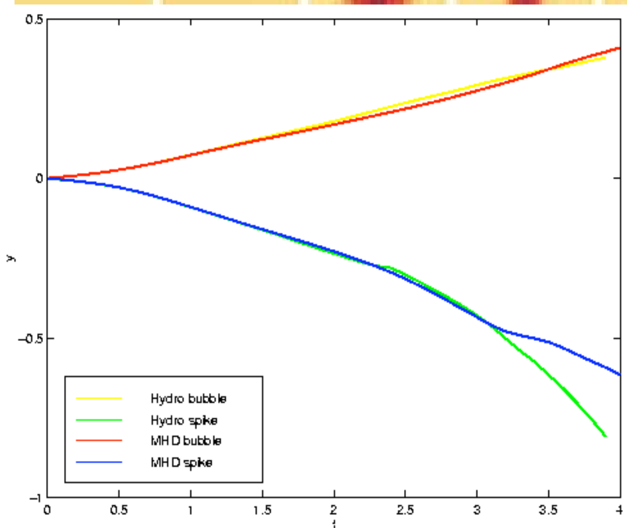


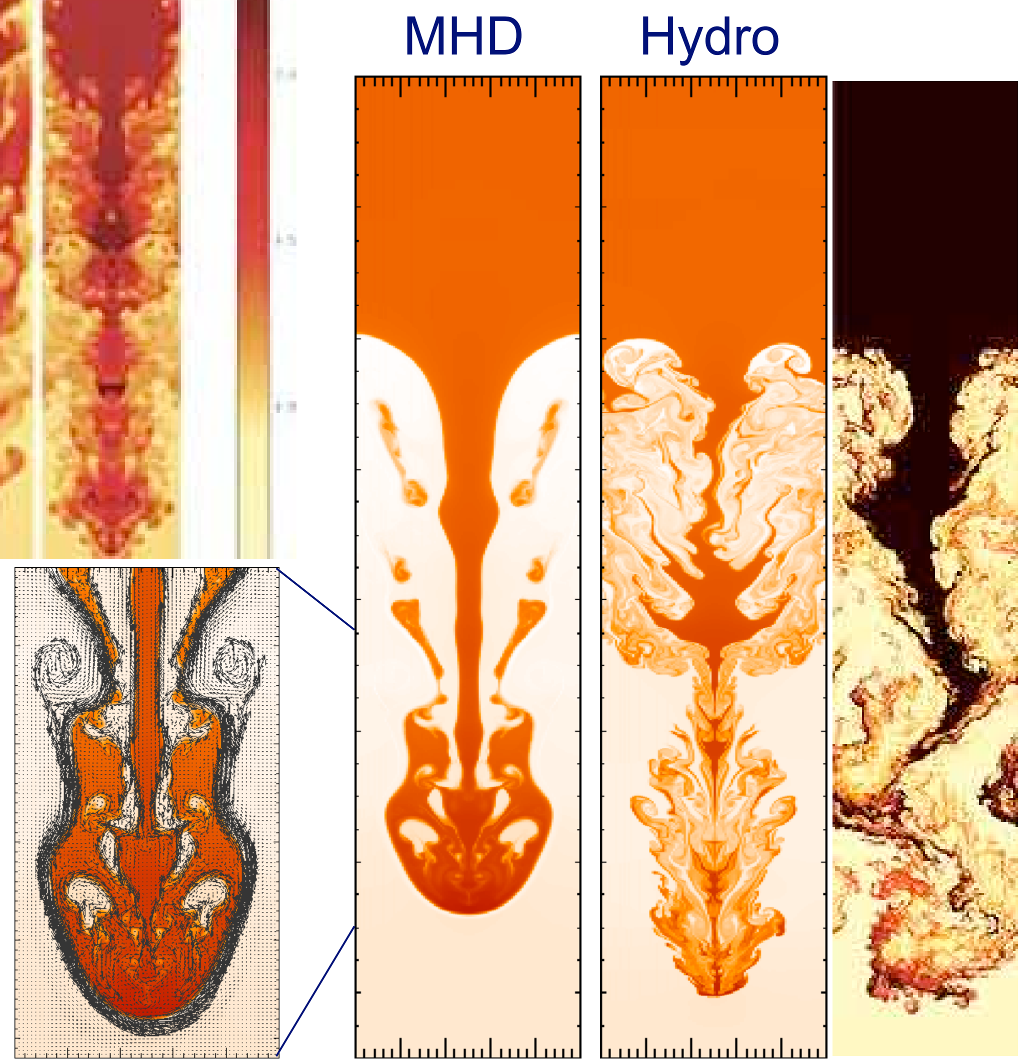
Rayleigh-Taylor



Rayleigh-Taylor Instability







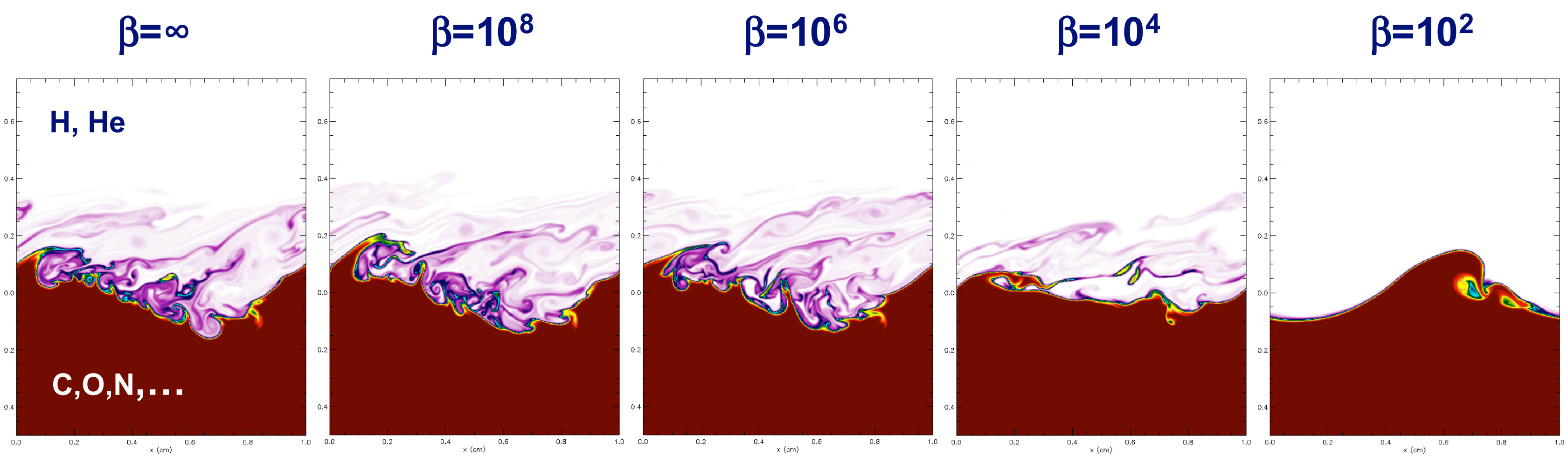
Surface Gravity Waves

Possible mechanism for heavy element enrichment in classical novae (Rosner et al, 2001; Alexakis, et al, 2003). Enrichment needed to explain observed energetics

Hydro models lead to desired enrichment but MHD models may not produce it

Initially weak field is amplified to equipartition above the mixing layer; the amplified field suppresses the instability and mixing of material at the interface

Precise details are sensitive to relative strength of field and gravity and the amount of magnetic field reconnection in the shear layer



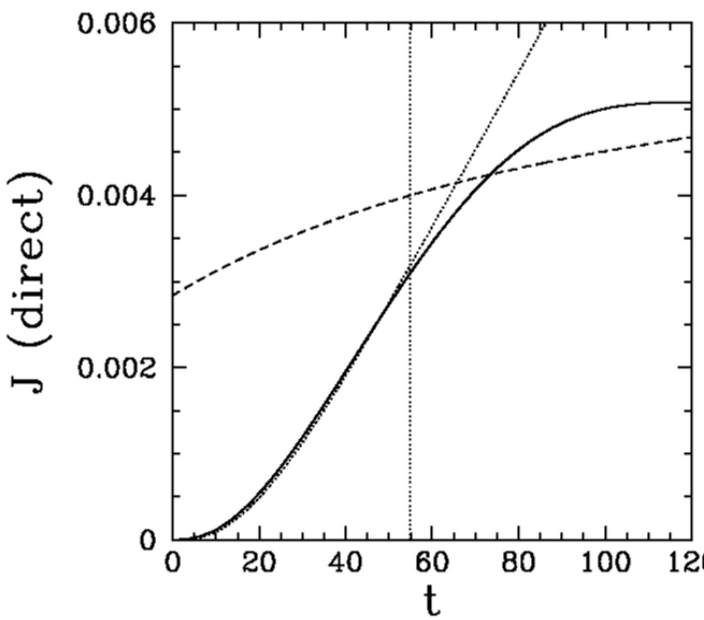
Wave Driven Taylor Problem

CMRS collaboration (Fitzpatrick et al, 2003)

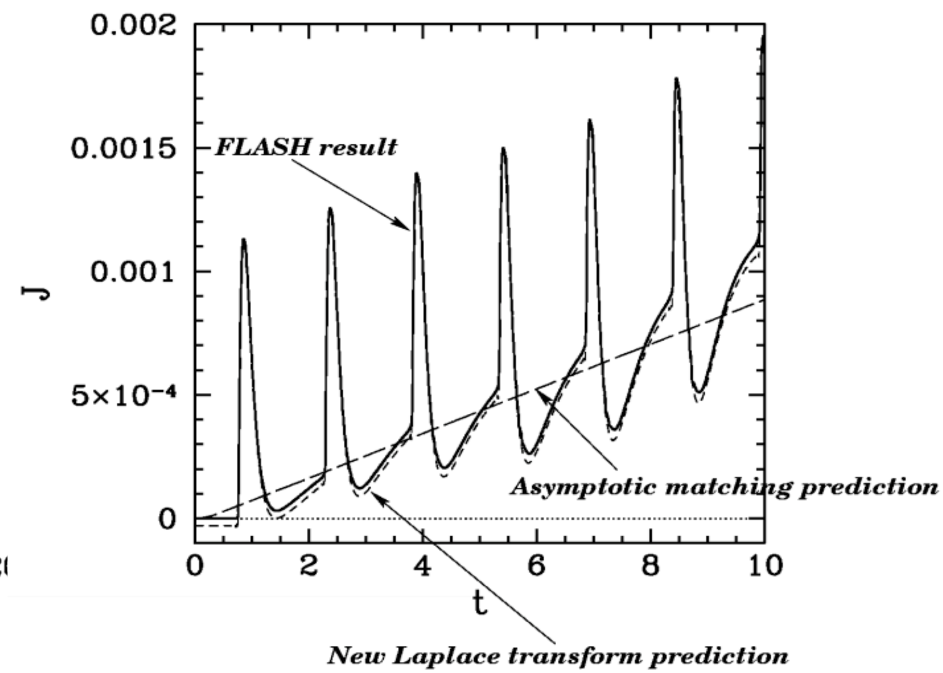
Response of a stable slab plasma equilibrium to applied wall perturbations

Analytic Laplace transform theory developed and compared with numerical results

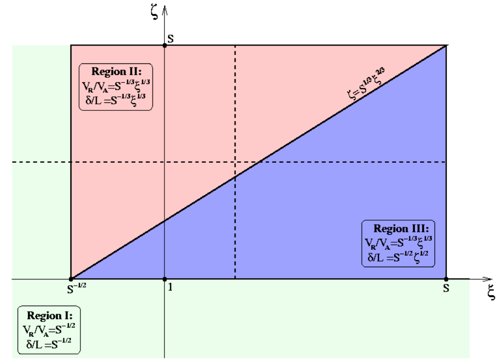
Slow perturbation



Rapid perturbation



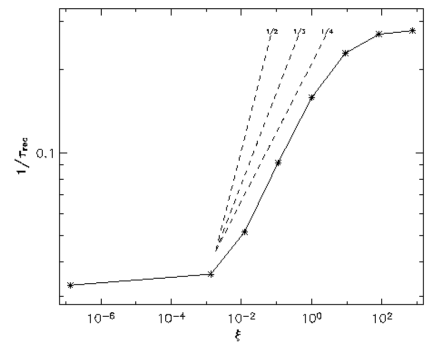
Reconnection in Rosette Configuration



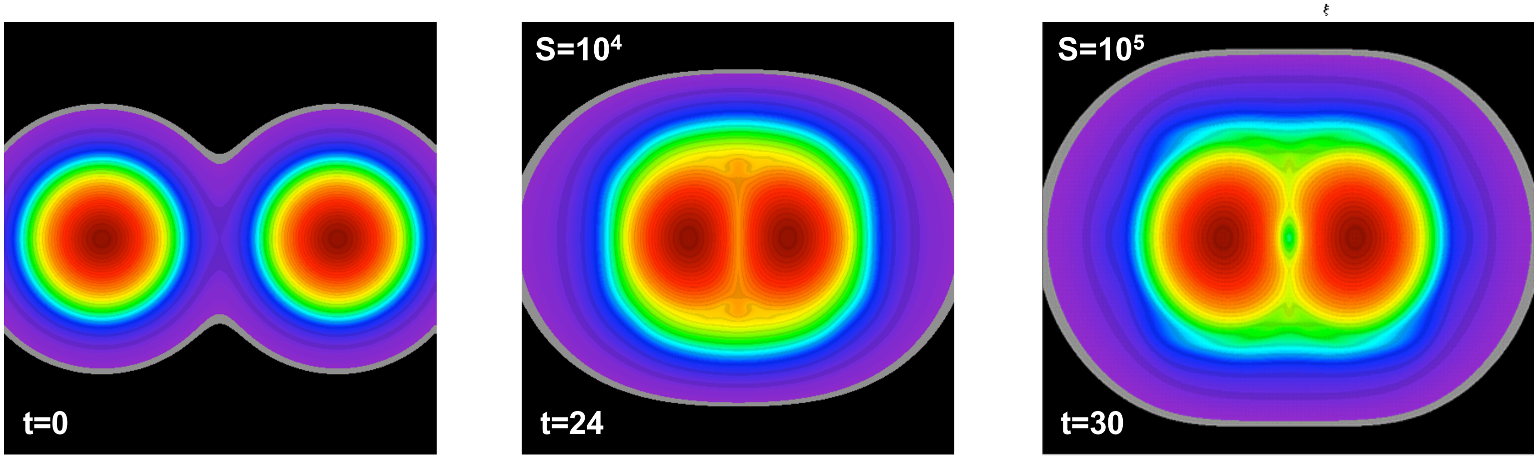
Verified Kulsrud's (2001) Sweet-Parker v. Petchek theory with anomalous resistivity (e.g. due to lower hybrid instability):

$$\eta(j)=\begin{cases}\eta_0,\ j<j_c\\ \eta_0+\eta_*\dfrac{j-j_c}{j_c},\ j>j_c\end{cases}$$

$$\frac{V_R}{V_A}=\left(\frac{\delta_c}{L}\frac{1}{S_*}\right)^{1/3}\qquad \delta_c=B_0/4\pi j_c\qquad S_*=\frac{V_A L}{\eta_* c/4\pi}$$



Theory seems to hold (Malyshkin, Linde, Kulsrud, 2004) provided that the reconnection layer remains stable on large scales



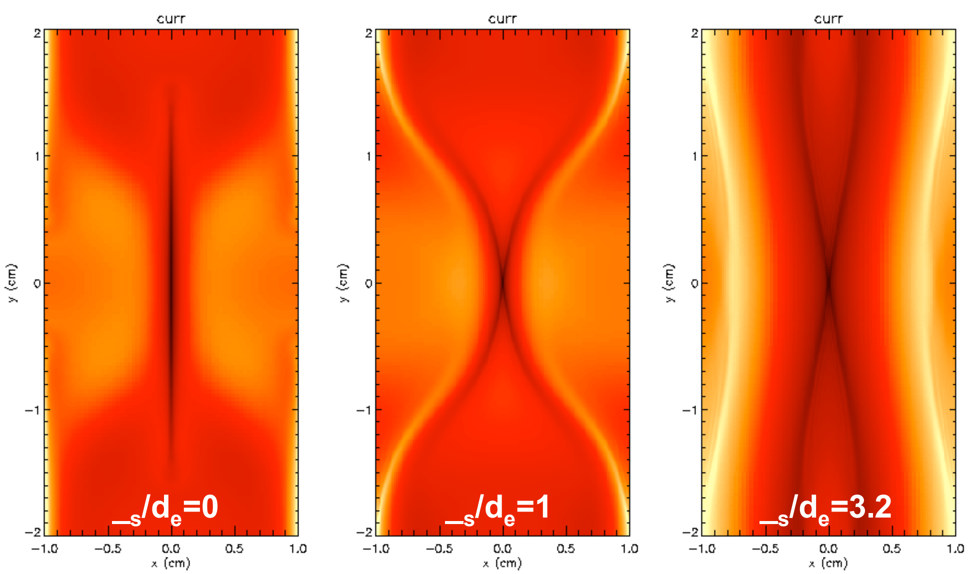
Reconnection in 2D Hall MHD

CMRS project with A. Bhattacherjee and K. Germaschewski to implement and analyze the model of Grasso et al., 1999
Model includes effects of electron inertia and compressibility

$$
\begin{aligned}
\frac{\partial F}{\partial t} + [\phi, F] &= \rho_s^2 [U, \psi] \\
\frac{\partial U}{\partial t} + [\phi, U] &= [J, \psi] \\
F &= \psi + d_e^2 J \\
J &= -\nabla^2 \psi \\
U &= \nabla^2 \phi \\
\vec{B} &= B_0 \hat{z} + \nabla\psi \times \hat{z} \\
\vec{v} &= \hat{z} \times \nabla\phi
\end{aligned}
$$

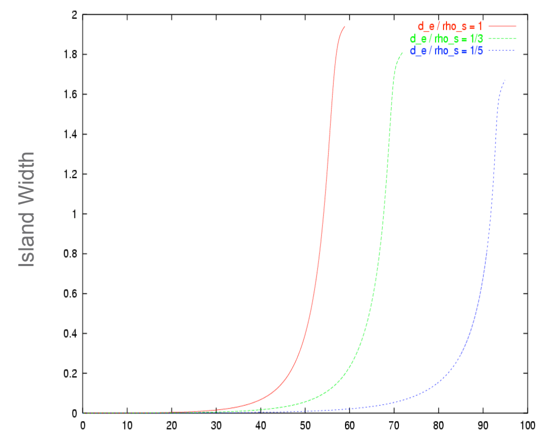
The critical parameter is $\_{s}/d_e$, where $\_{s}$ is the ion sound Larmor radius and d_e is the inertial skin depth. If $\_{s}/d_e > 1$, ions and electrons decouple, vorticity and current layers split into different layers, and fast reconnection pattern develops.

Nonlinear growth rate depends on d_e and δ_0 in finite time



Island width growth rate

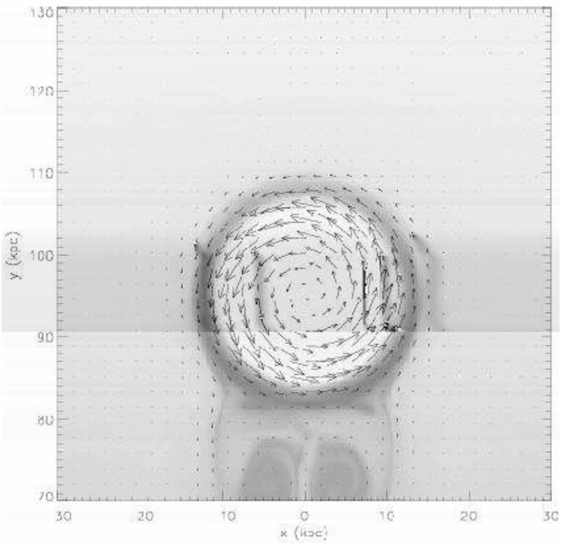
$$\gamma_L^{-1} = 1 / k \rho_s^{2/3} d_e^{1/3}$$



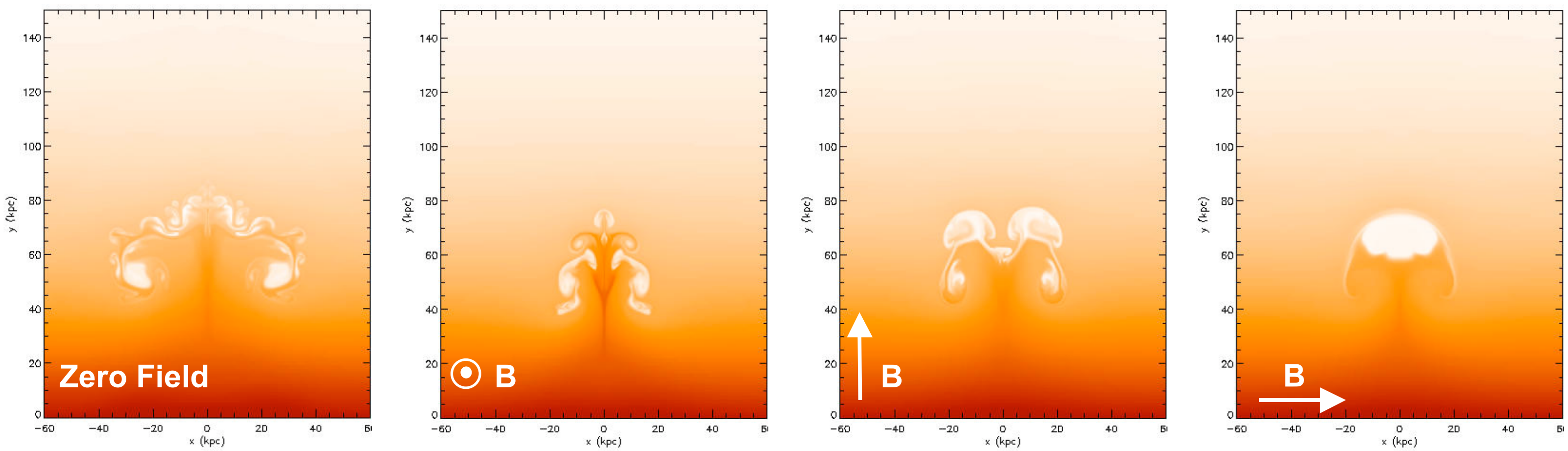
Magnetic Bubbles on Galactic Scale

Recent *Chandra* and *XMM-Newton* observations in X-rays showed emission voids up to 30 kpc in size
Voids coincide with regions of synchrotron emission

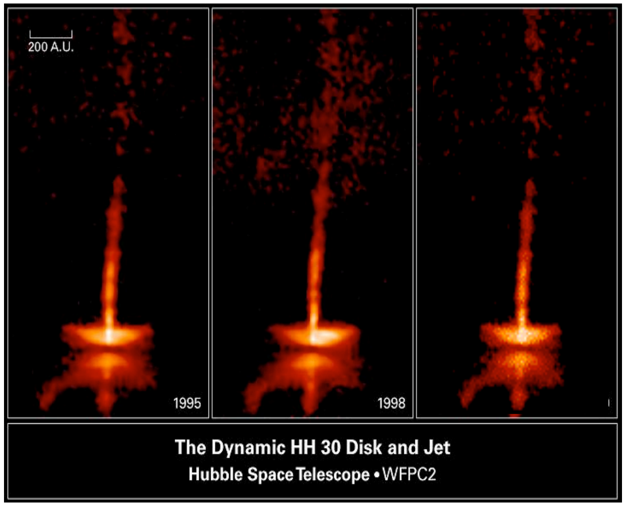
Robinson et al, 2004, and Brüggen and Kaiser, 2002, confirm that bubbles without magnetic field are torn apart by shear as they rise; even weak magnetic field prevents bubble breakup; internal magnetic field provides best support against breakup; magnetic field at the edge of a bubble may be needed to inhibit thermal diffusion of the bubble



Parameters : $\rho = 10^{-26}$ g/cm^3 , $\beta = 10^2$, time = 280 Myr , density contrast = 10 : 1



Jet Launching From Resistive Accretion Disk



Magnetically driven accretion and jet acceleration:

Poloidal field extracts angular momentum from the disk and transfers it to the outflow material: magneto-centrifugal mechanism (Blandford & Payne, 1982)

Lorentz forces accelerate the outflow

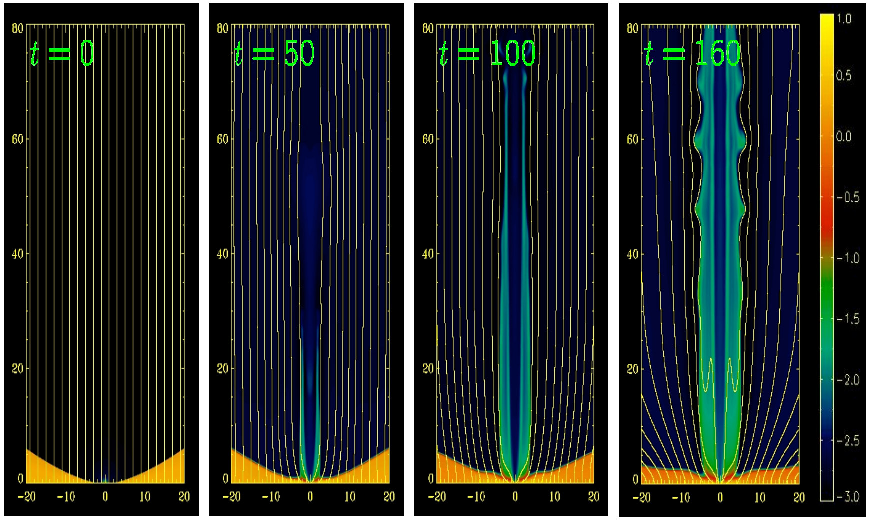
Generated toroidal field collimates the jet

Zanni et al, 2003 (Torino collaboration):

Cylindrical version of the MHD module

Produced highly collimated jet starting from a resistive Keplerian disk without forcing accretion

Demonstrated that accretion and collimation of the jet is driven by magnetic “hoop stress”



Accretion Onto a Compact Object

In a polar system (B>10^7 G) accretion disk is disrupted and the overflowing matter is collimated by the magnetic field into a column that falls on the compact object poles

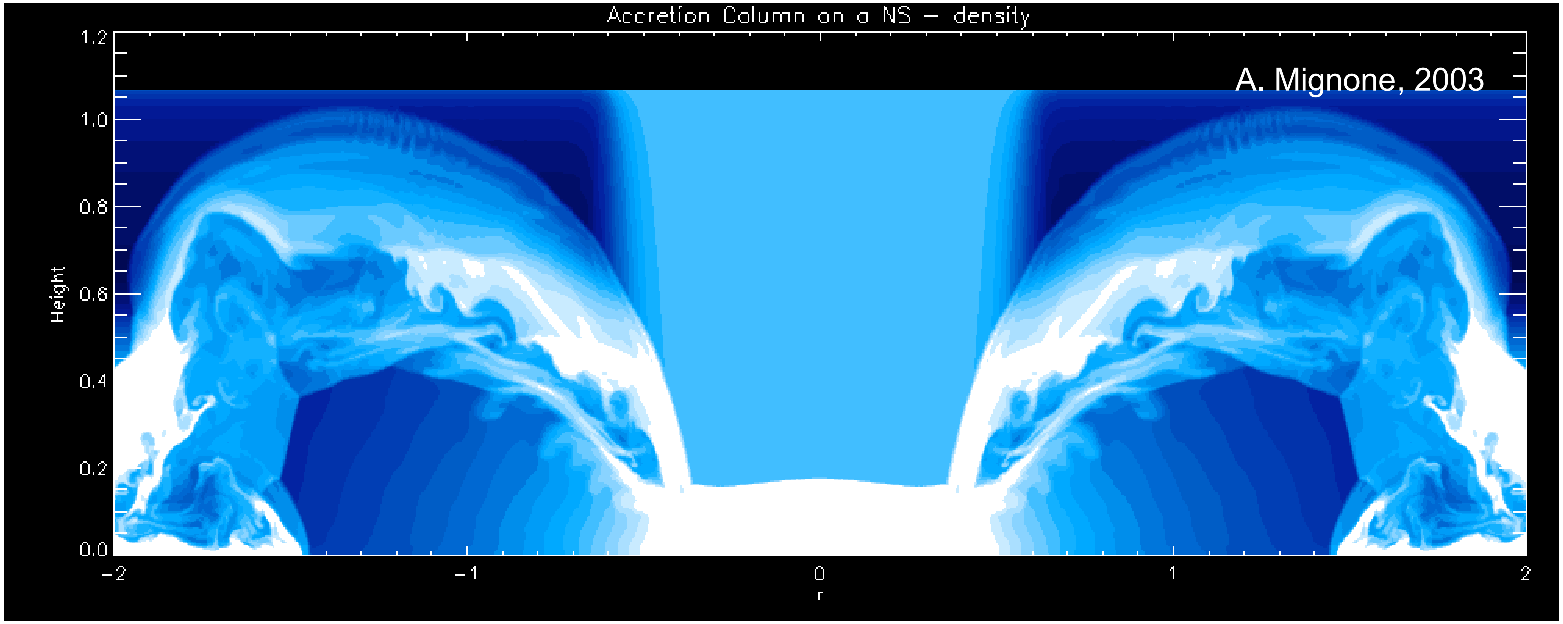
How strongly is flow confined? Does spreading occur? What instabilities are present?

Mignone, 2003: Strong fields completely collimate the flow and no spreading occurs

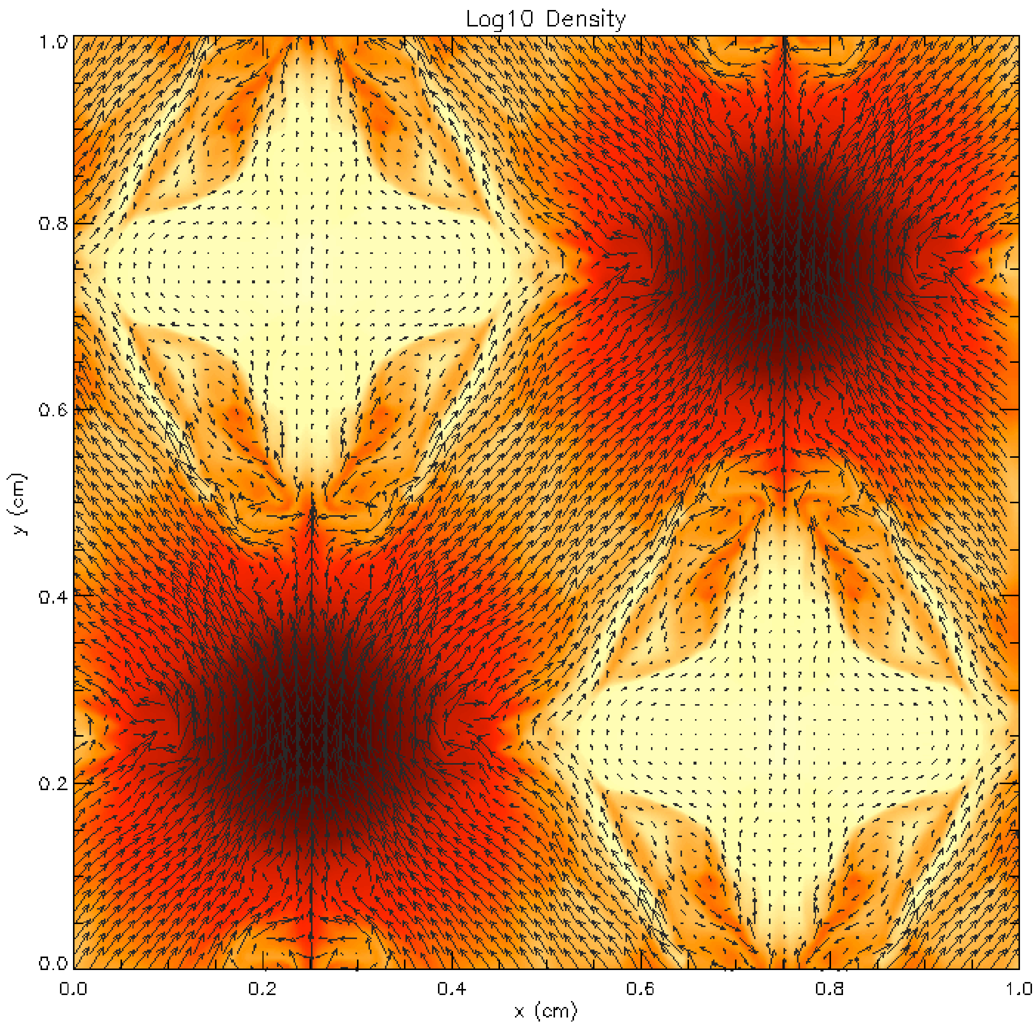
1-D models confirm that the flow is thermally unstable with frequencies ~ Hz

2-D models do not show thermal instabilities; magnetic tension provides damping

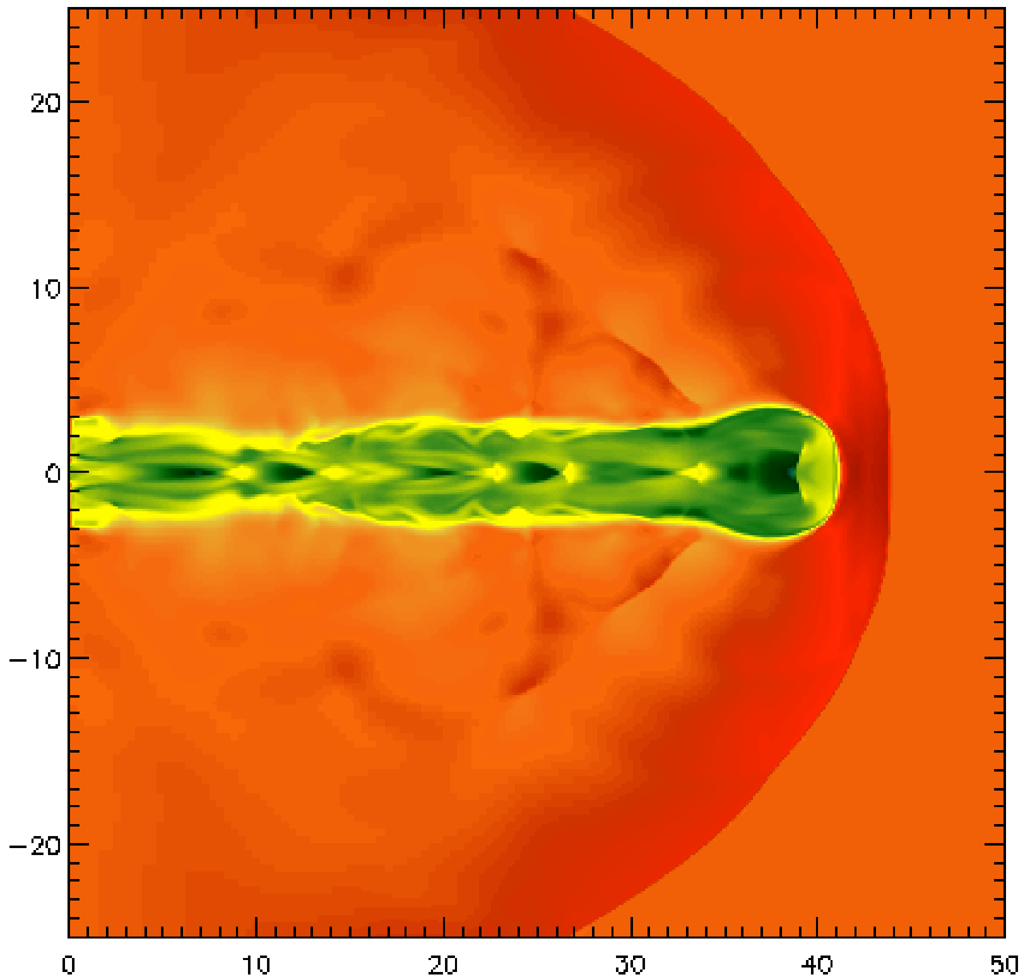
All models are very sensitive to boundary conditions and cooling functions



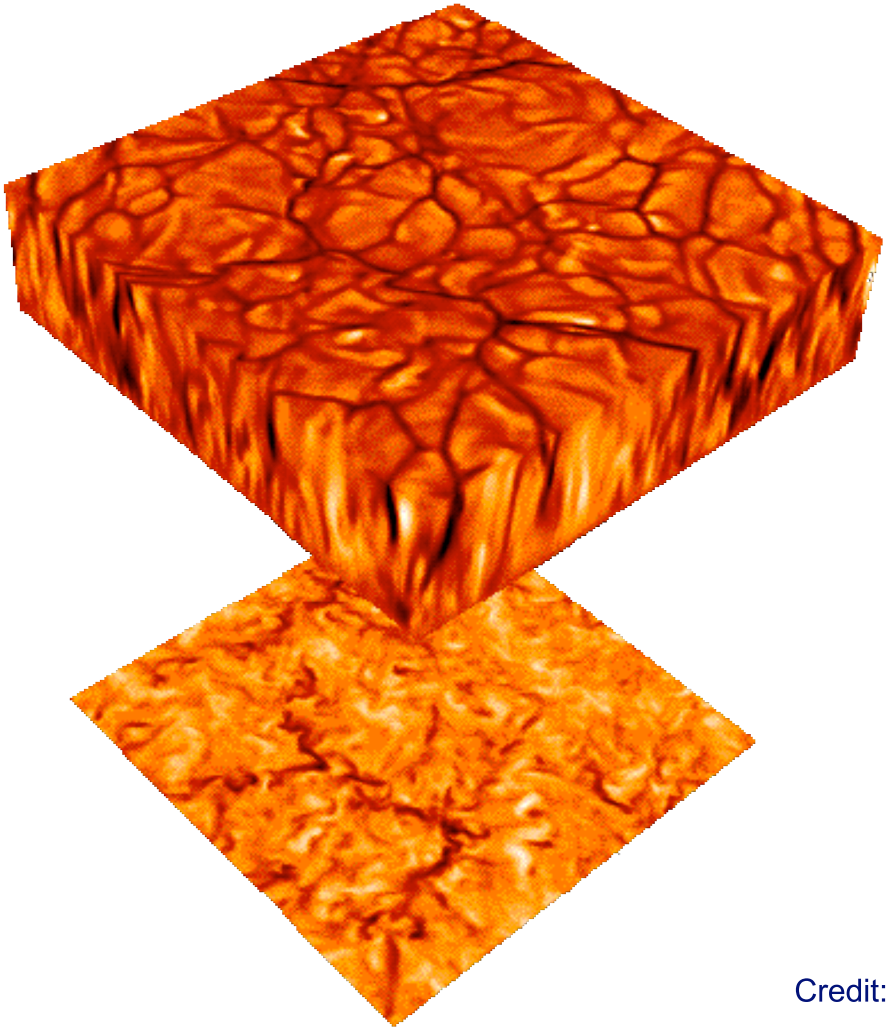
Self-Gravitating Plasmas



Relativistic Jets

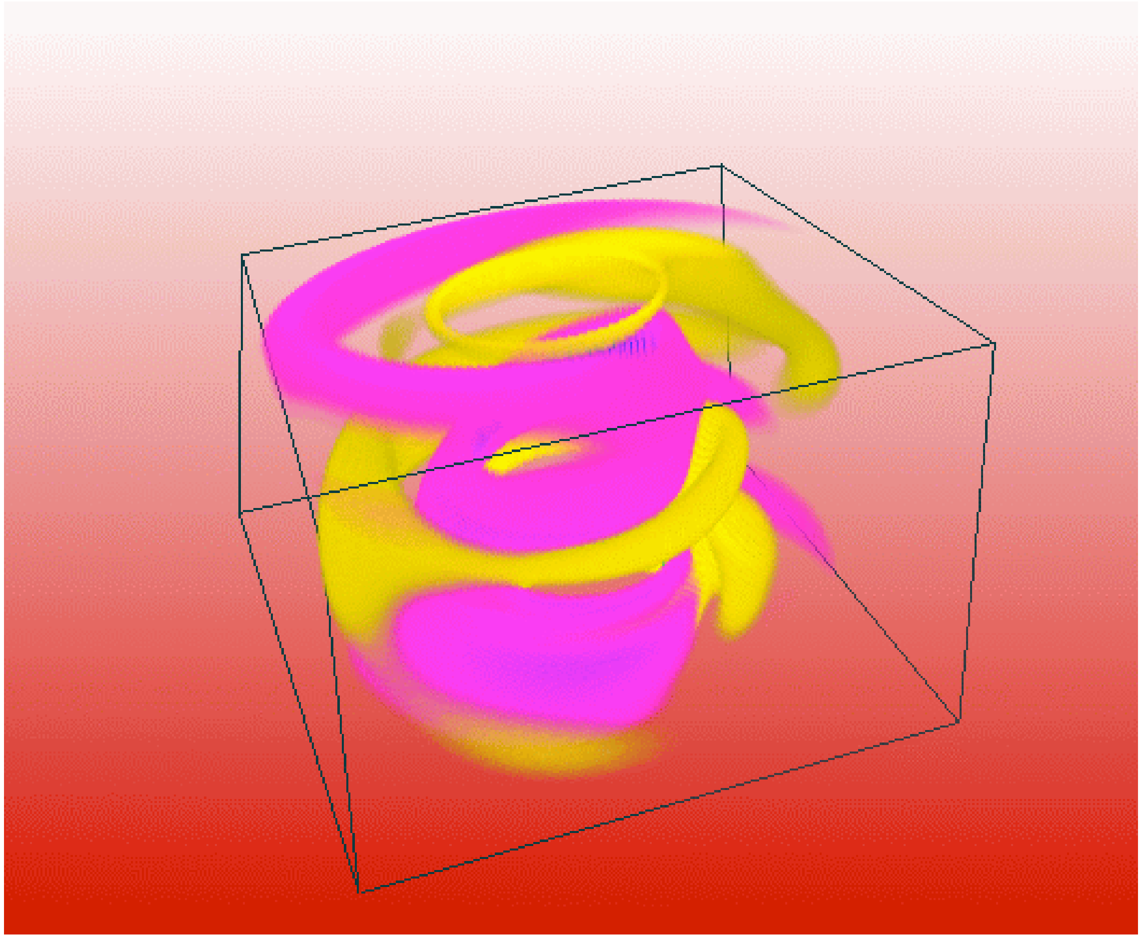


Solar Convection and Dynamo



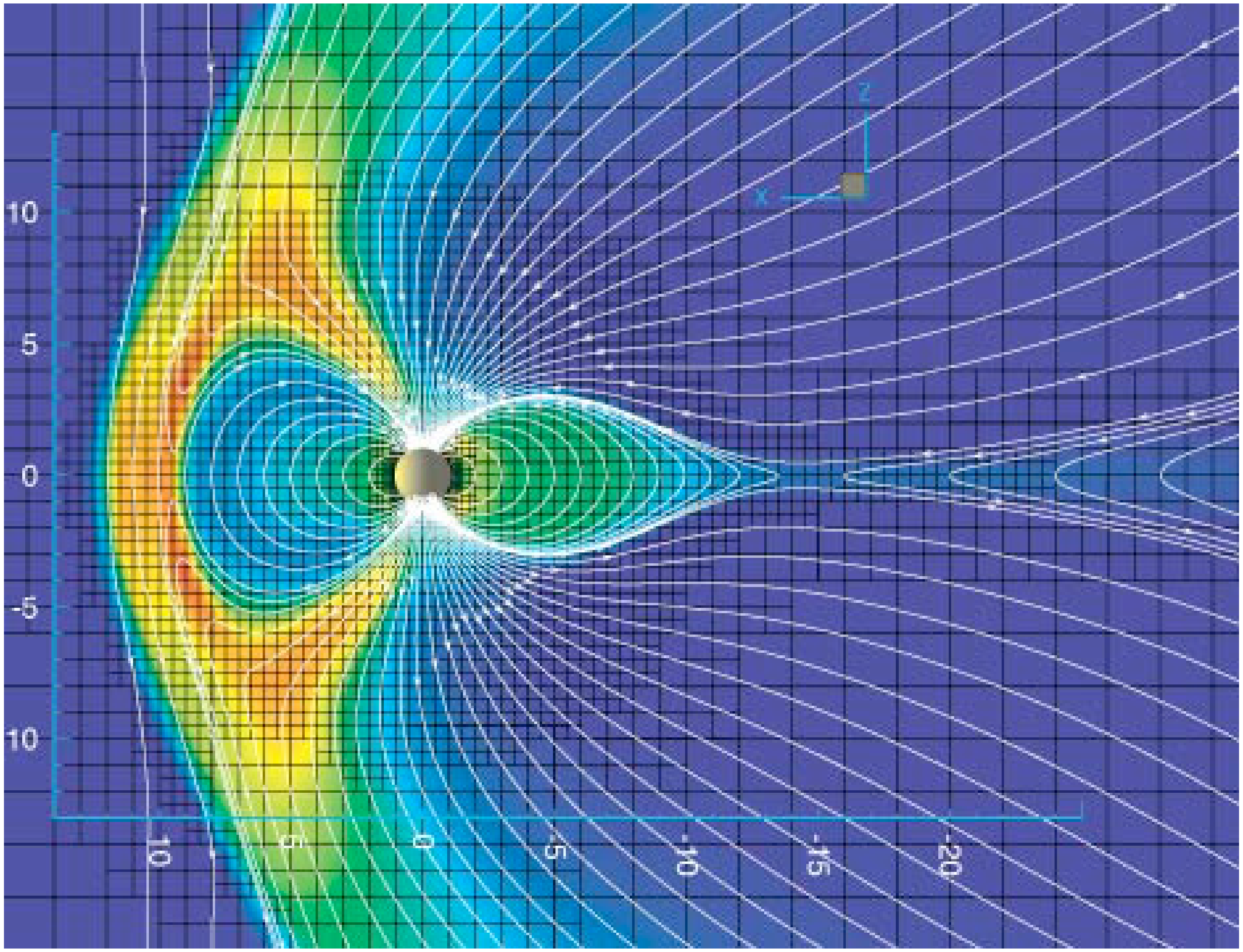
Credit: Cattaneo

Magnetorotational Instability (MRI)



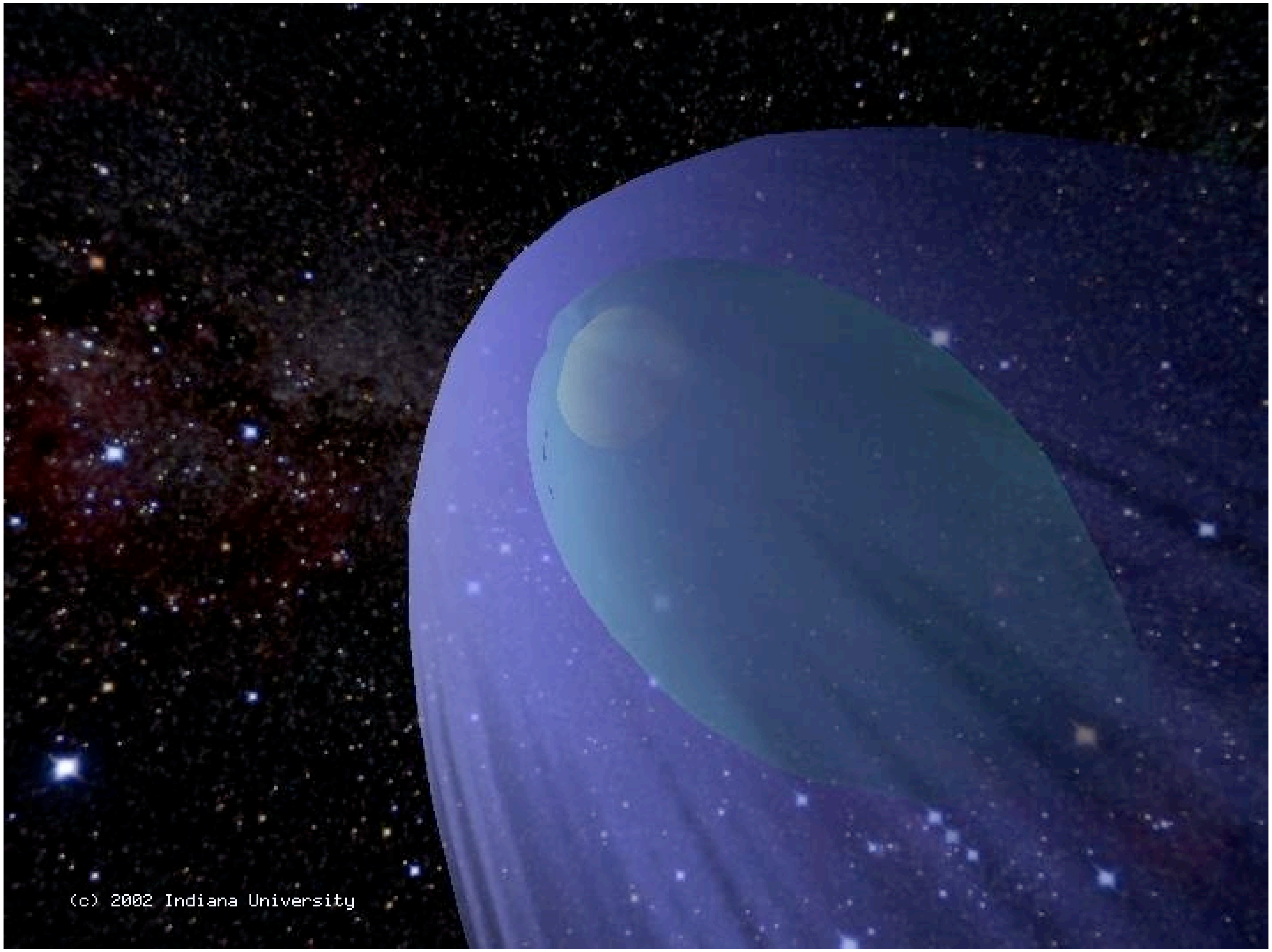
Credit: Obabko and Cattaneo

Magnetospheres



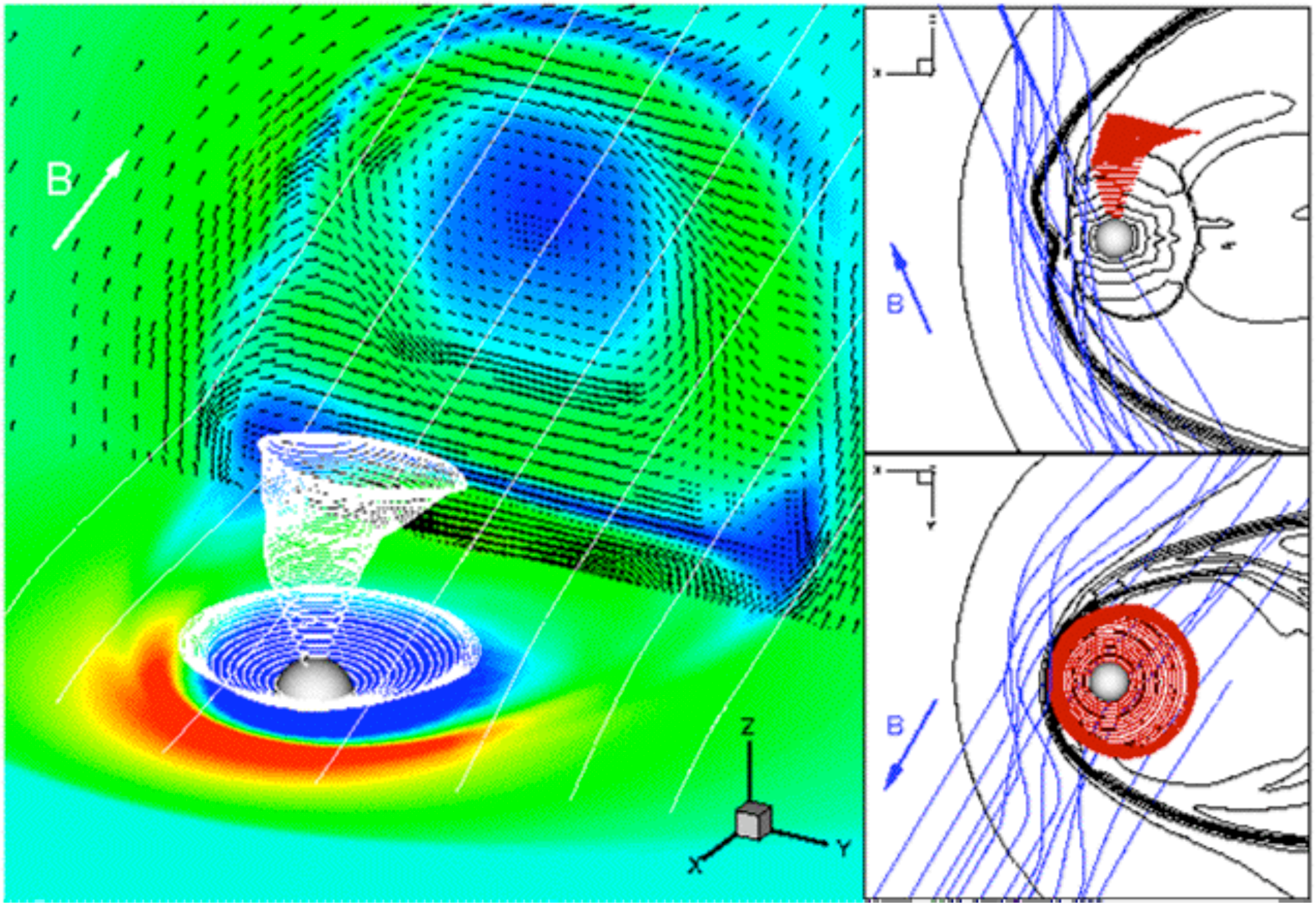
Credit: DeZeeuw, Gombosi et al, BATS-R-US

Heliosphere



Credit: Hanson, Frisch, Linde

Heliosphere



Credit: Linde

Resources

- Books:
  - Magnetohydrodynamics, A. Jeffrey
  - Nonlinear Magnetohydrodynamics, D. Biskamp
  - Waves in Plasmas, T. Stix
  - Magnetohydrodynamics, A. Kulikovskii
  - Plasma Electrodynamics, A. Akhiezer
  - NRL Plasma Formulary
  - Classic and recent literature in your field
  - Books on software design and (software) project management

- Papers:
  - Roe, Powell, Gombosi, Colella, Balsara, Tóth, Dai and Woodward, Del Zanna, Komissarov

- People:
  - Talk to as many experts as you can. It never hurts.

Questions?

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